Tutorial on

Modeling and Analysing Images of Generic Cameras

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Contents

- Introduction
- General imaging models
- Non-parametric calibration and distortion correction
- Non-parametric self-calibration
- Structure-from-motion

Introduction

There exist lots of camera designs:

Introduction

Some applications:

Automatic Vehicle Navigation

Aerial Mosaics

3D Video Conferencing

Surveillance

Shape Computation

Panoramic Imaging

- Many applications require/benefit from a specific type of imaging system
- Work underlying this tutorial started by considering omnidirectional systems (large field of view)

Introduction

Videoconferencing:

Introduction

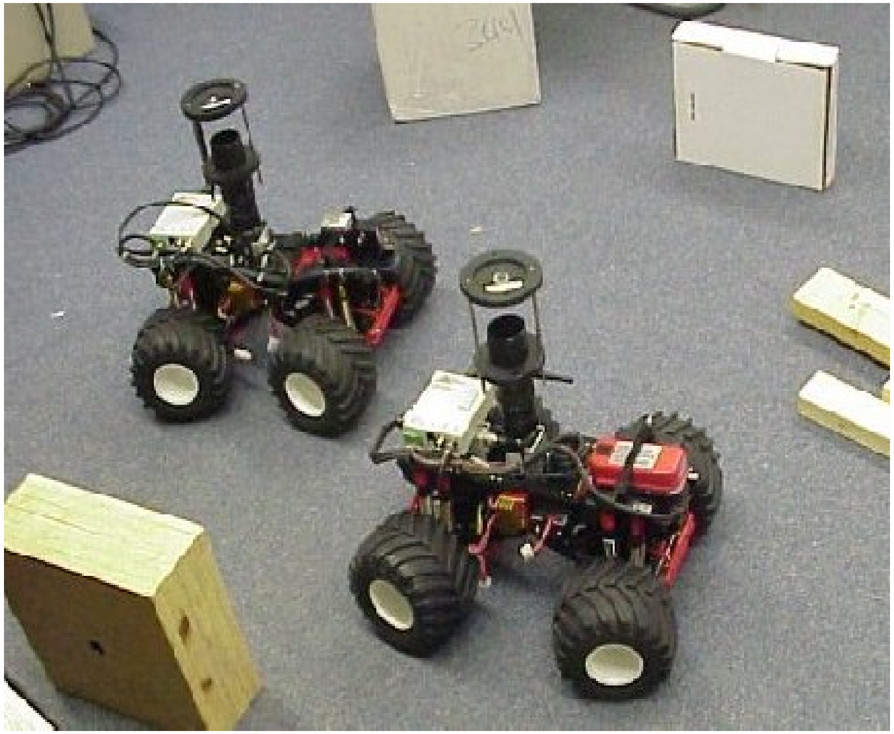
Surveillance:

Introduction

Surveillance:

Introduction

Robot navigation (including obstacle avoidance):



Taylor et al. – GRASP

Santos Victor et al. – ISR/IST

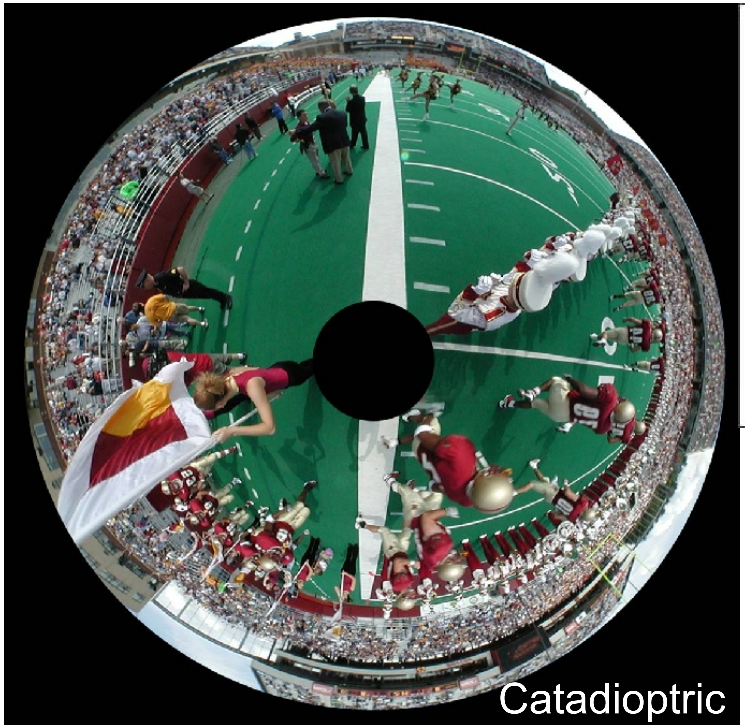
Introduction

Panoramic imaging, here mosaicing:

Problematic for dynamic scenes:

Introduction

Panoramic imaging with omnidirectional cameras:

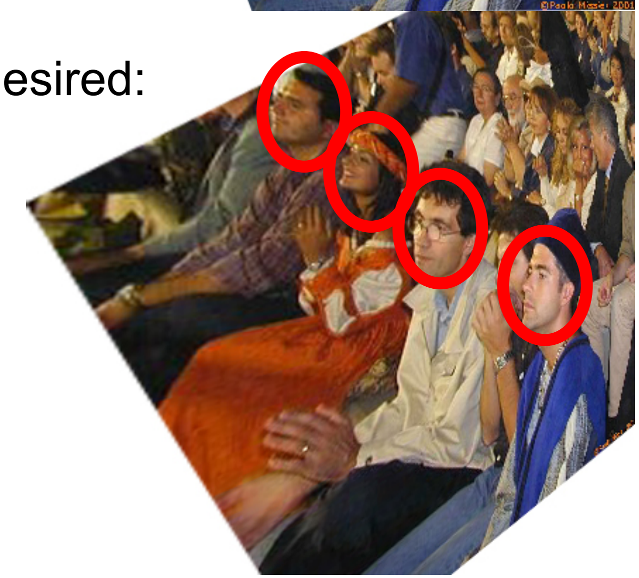


Introduction

Design of tailor-made imaging systems:

Usual:

Desired:



Introduction

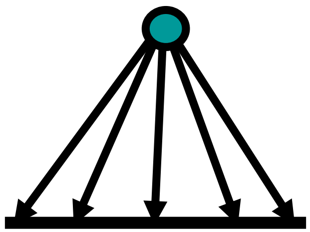
Design of tailor-made imaging systems:

By Julian Beever

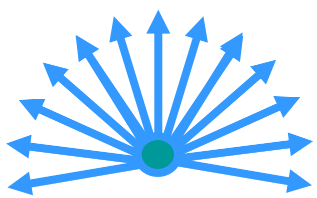
Introduction

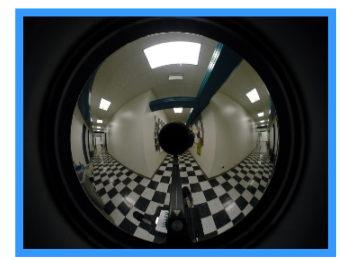
Different cameras "sample light rays" in different ways:

Perspective cameras:

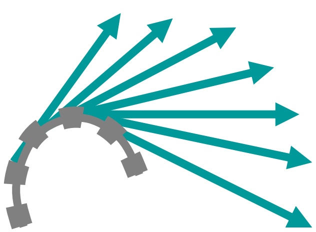
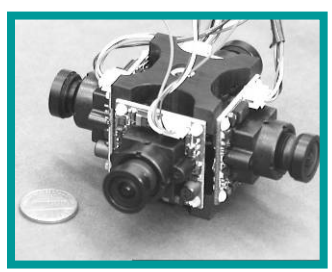


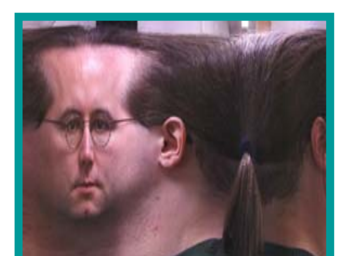

Single viewpoint cameras:



Non-single viewpoint cameras:



Introduction

Each camera type comes with a particular model and often, particular calibration and structure-from-motion algorithms

Main motivations for my related works:

- Propose generic camera models and calibration algorithms
- Highlight common principles underlying structure-from-motion algorithms for different camera models
- Generalize (parts of) the structure-from-motion theory, e.g. multi-view geometry (epipolar, trifocal and quadrifocal geometry)

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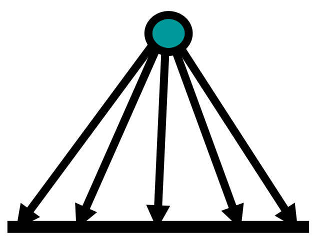
Imaging Models

Perspective cameras:

- Imaging model well-known…
- Interior orientation (intrinsic parameters)
  allows to perform **projection**: 3D points → image points)
  and **back-projection**: image points → projection rays (lines of sight)




Imaging Models

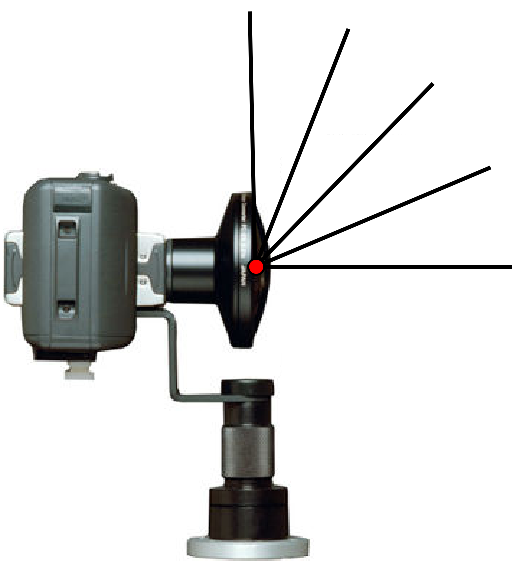
Single viewpoint cameras:

- Perspective projection plus radial or decentering distortion
  - imaging model well-known…
  - again, interior orientation (intrinsic parameters) allows to perform **projection** and **back-projection**
  - calibration approaches:
    - plumbline calibration: use images of straight line patterns to estimate “non-perspective” parameters
    - calibration with control points: compute all parameters of the model using bundle adjustment

Imaging Models

Single viewpoint cameras:

- Fisheyes
  - several models have been proposed (ad hoc or derived from actual lens designs)
  - e.g. equi-angular model (existence of distortion center and optical axis such that distance of image point to distortion center is proportional to angle between projection ray and optical axis)



Imaging Models

Catadioptric systems (camera + mirror):

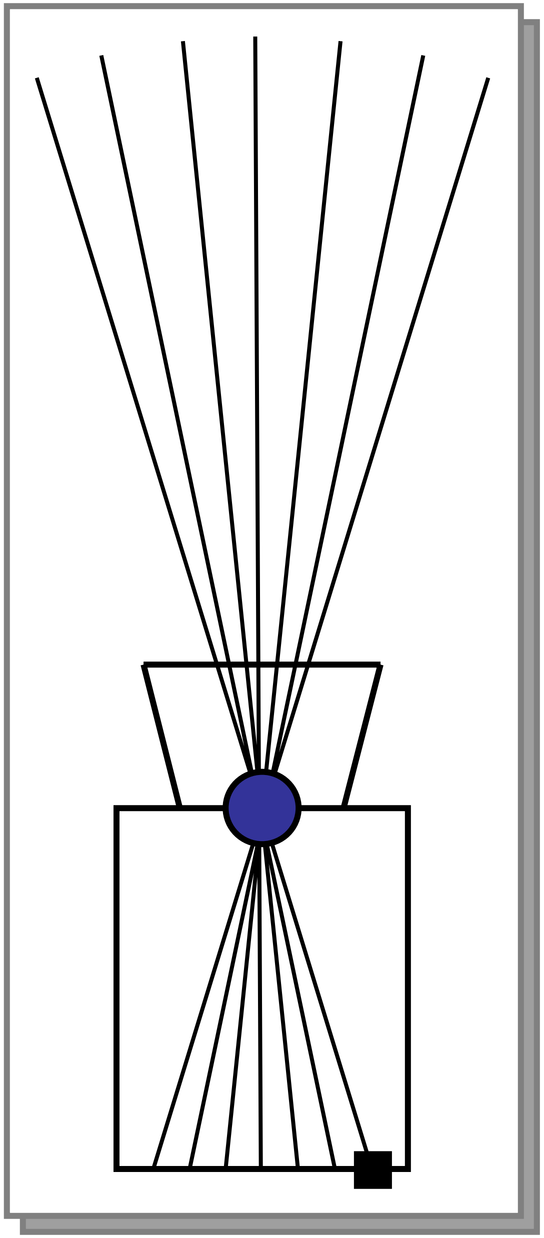
- Knowledge of mirror shape and position relative to camera, together with camera's interior orientation, allows to perform back-projection

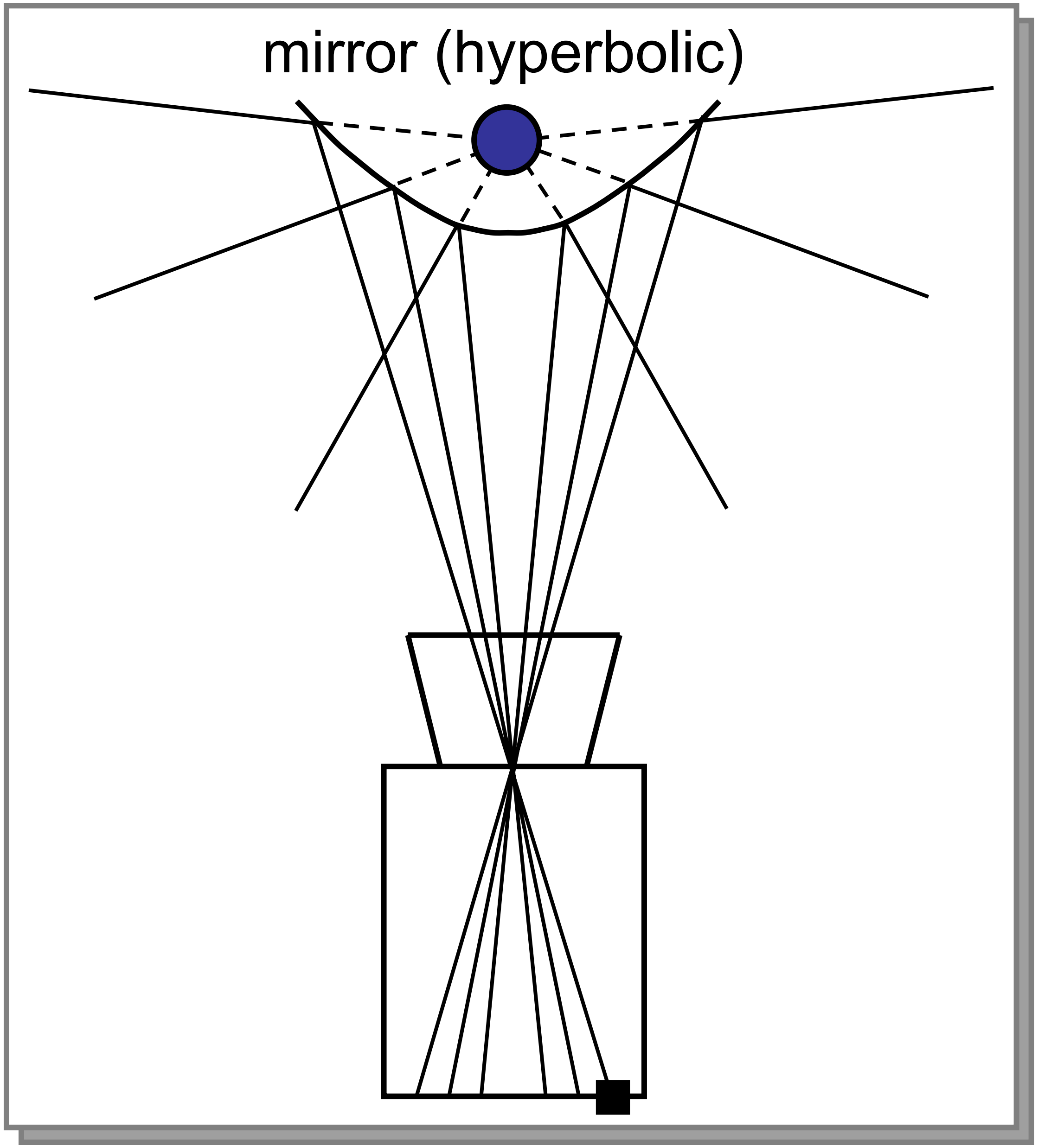
Imaging Models

Back to single viewpoint cameras:

- **Central** catadioptric systems
  - with appropriate mirror shape and position, system has a single effective viewpoint (cf. next slide)
  - practically relevant: parabolic mirror + orthographic camera, hyperbolic mirror + perspective camera
  - various imaging models have been proposed:
    - models whose parameters represent correlations between mirror shape/position and interior orientation of camera
    - unifying models for all types of central catadioptric cameras
  - calibration approaches:
    - plumbline approaches (sometimes with closed-form solutions)
    - calibration with control points: compute all parameters of the model using bundle adjustment

Imaging Models





Imaging Models

Single viewpoint cameras:

- Central catadioptric system using **multiple** planar mirrors and cameras (so-called Nalwa pyramid)

  - perspective camera + planar mirror
    ≡ perspective camera with effective optical center on the other side of the plane

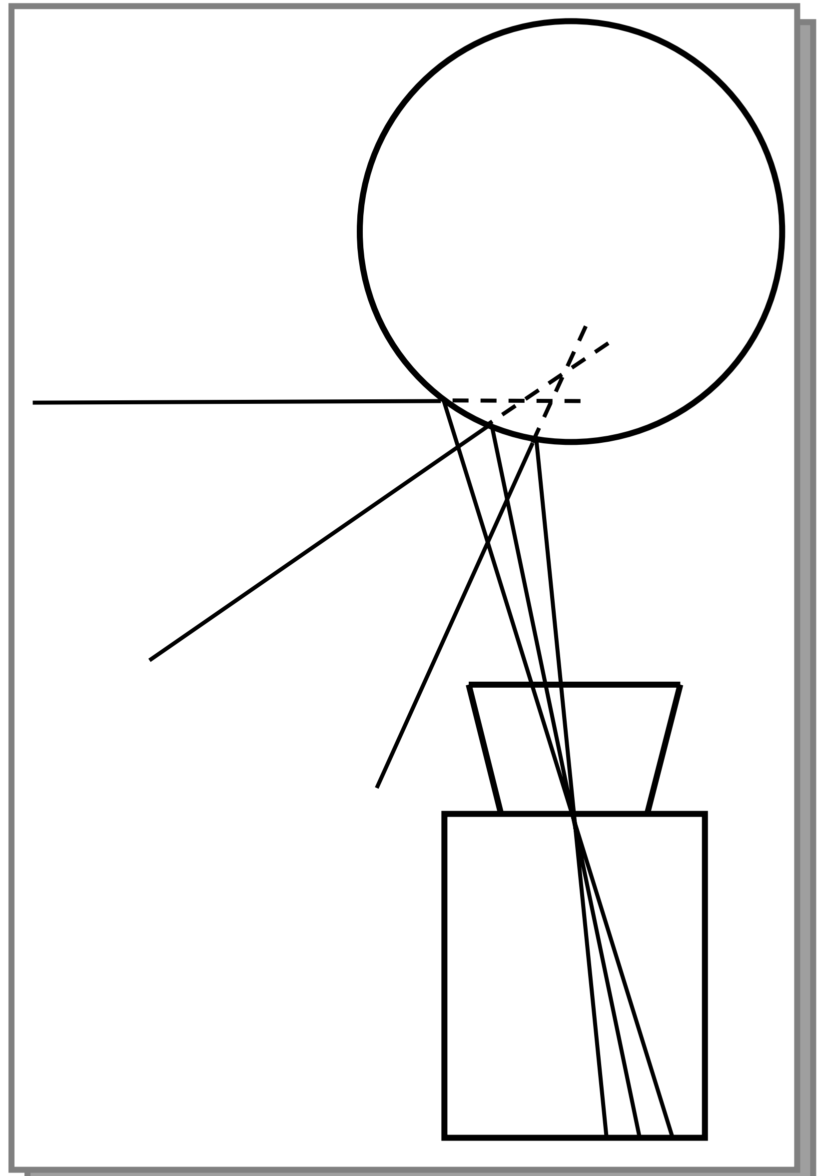
  - Nalwa pyramid: assemble pairs (camera, mirror) such that effective optical centers coincide

→ possibility to construct a high-resolution panoramic image

Imaging Models

Non-single viewpoint cameras:

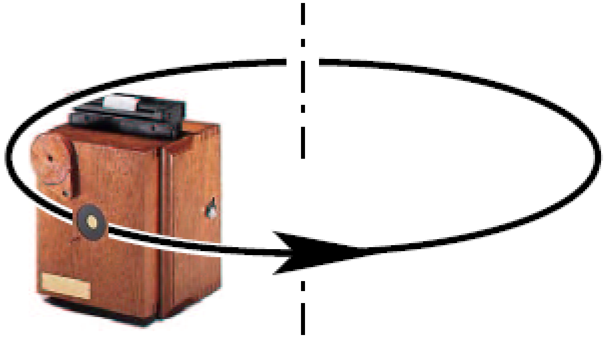
- **Non-central** catadioptric systems
  - spheres, cones or any non-quadric mirrors give non-central system: projection rays do not intersect in a single point
  - calibration approaches have been developed for individual systems
  - example:
    - mirror that leads to equi-angular imaging model

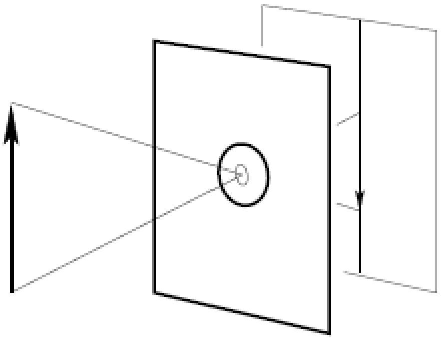


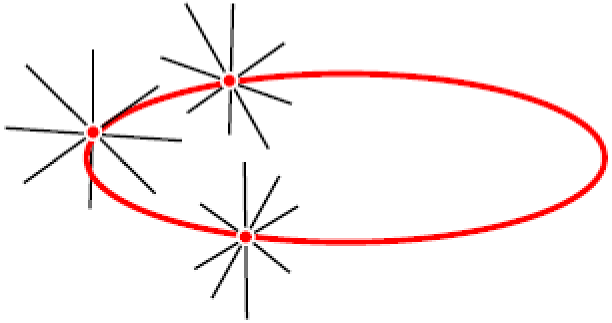
Imaging Models

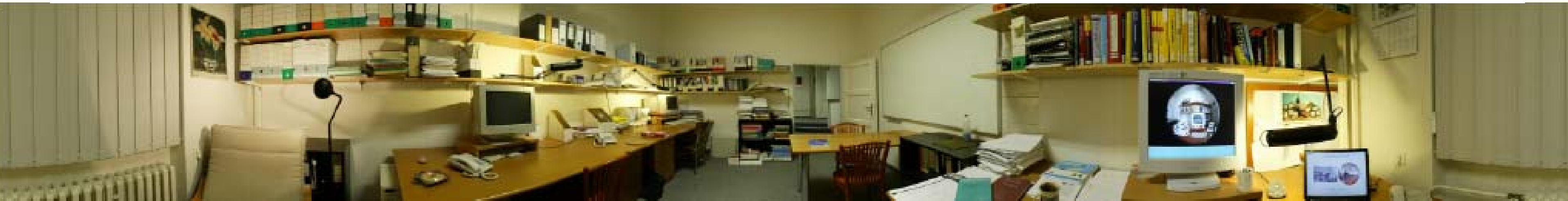
Other non-single viewpoint cameras:

- Pushbroom cameras
  - Moving linear camera acquires 1D images that are stitched together to a 2D image (motion is usually a lateral translation)
- So-called non-central mosaics
  - Acquired by a camera rotating about an axis not containing the optical center (from each image, take one or several columns of pixels and stitch them all together)









Imaging Models

Other non-single viewpoint cameras:

- So-called multi-perspective images
  - Acquired like a non-central mosaic but with camera looking inwards

Imaging Models

All above imaging models are subsumed by the following **generic imaging model**:

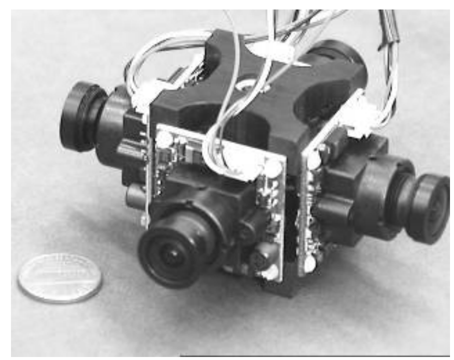
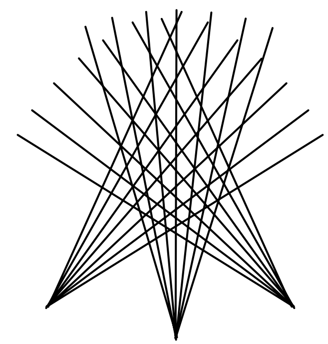
- A pixel “watches along” one viewing ray
- Camera model is lookup table, containing for each pixel the coordinates of the associated ray

Calibration = computation of all these rays

Imaging Models

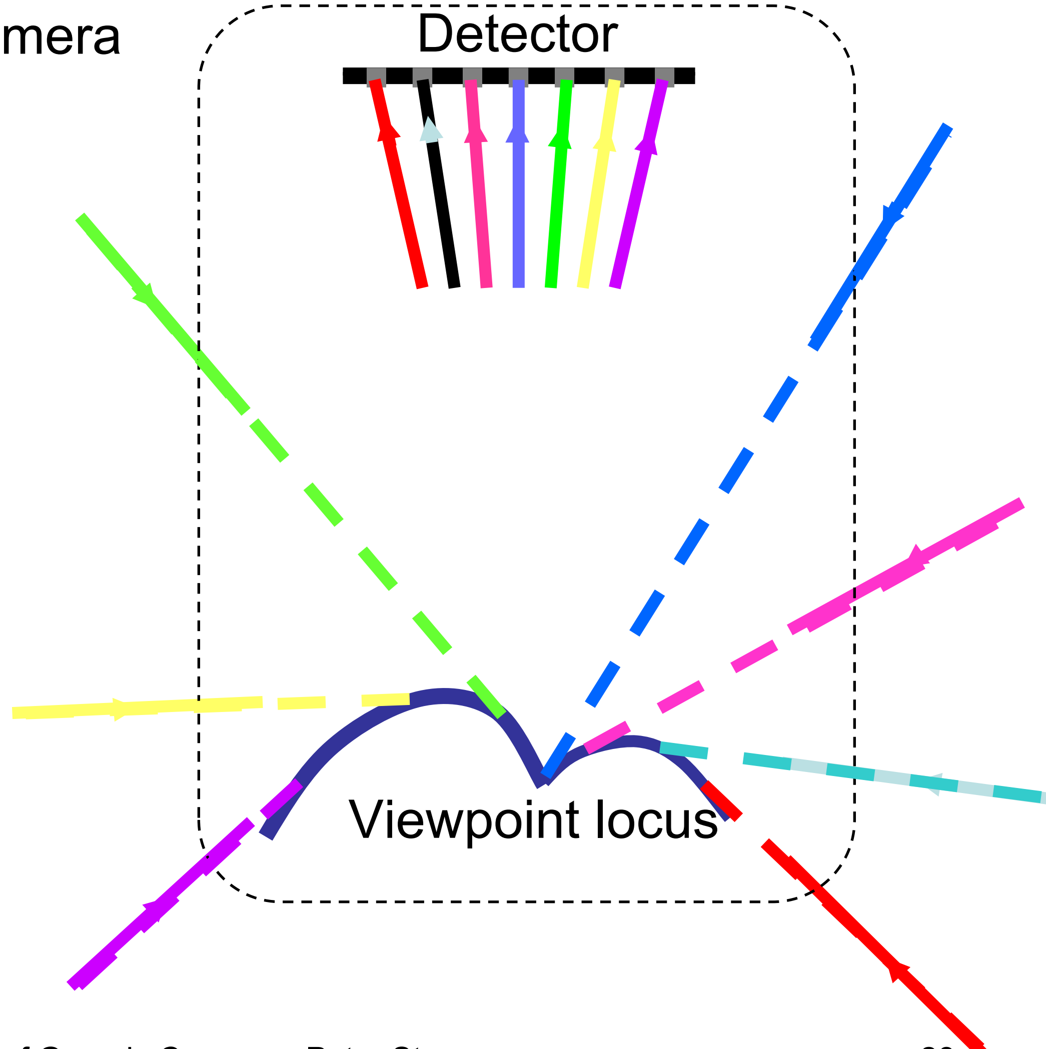
Comments on the generic imaging model:

- is idealized (in reality, a pixel sees more than a line)
- more complete model, including radiometric properties, is used by Grossberg and Nayar (ICCV 2001)
- other sampling than pixel-wise is possible (e.g. sub-pixel)
- conceptually, allows to consider a stereo or multi-camera system as a single camera: union of their pixels and associated rays

Imaging Models

Alternative model: caustic of a camera (surface touching all projection rays), also sometimes called viewpoint locus
(caustic of a single viewpoint camera is a single point)



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Non-parametric calibration — Basic idea

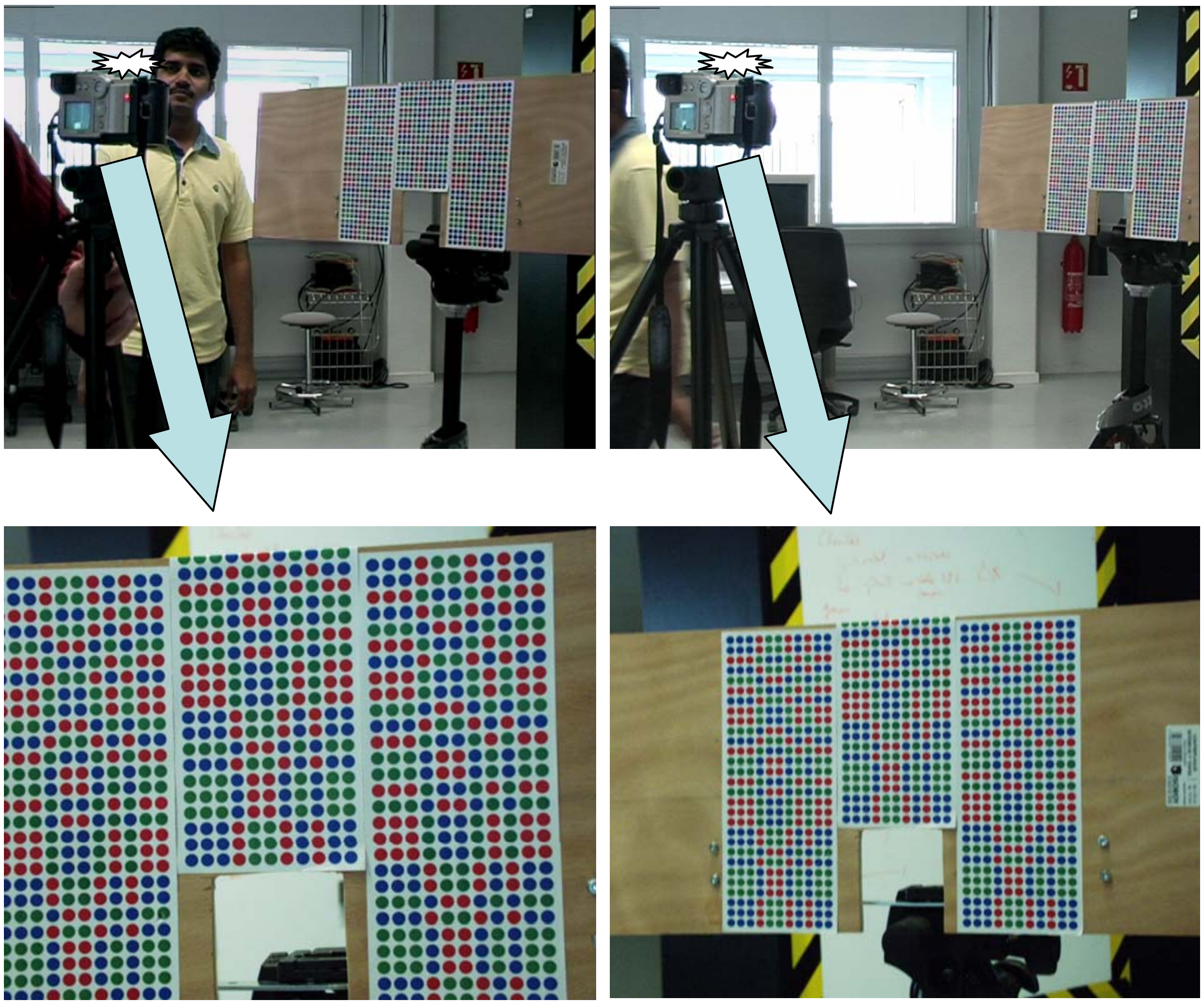
Input: images of calibration objects

Goal: compute projection ray for each pixel, in some 3D coordinate system

- General approach applicable for non-central cameras
- Variants for special cases (central and axial cameras)

Non-parametric calibration — Basic idea

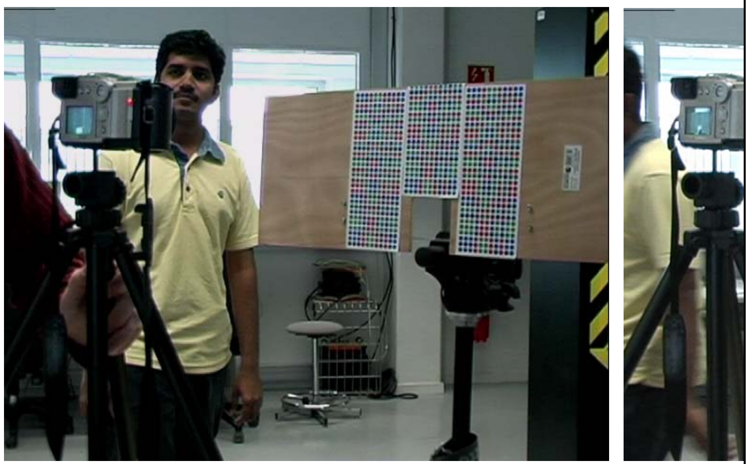
Approach using known motion: [Gremban-etal-ICRA'88,Champleboux-etal-ICRA'92, Grossberg-Nayar-ICCV'01]

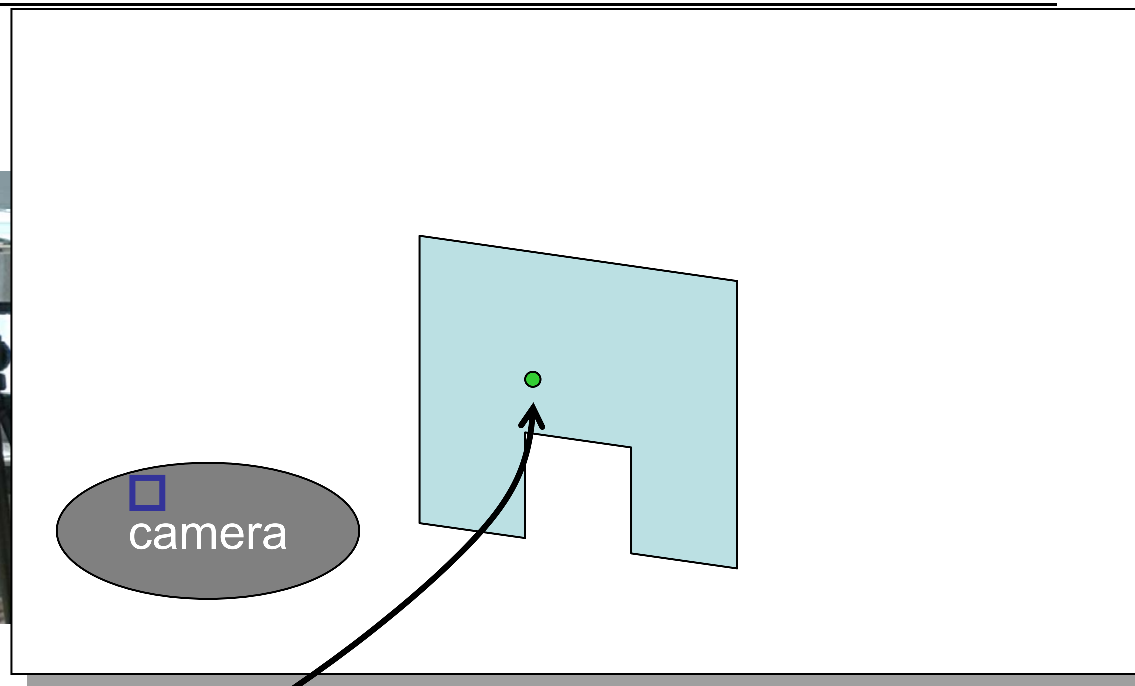


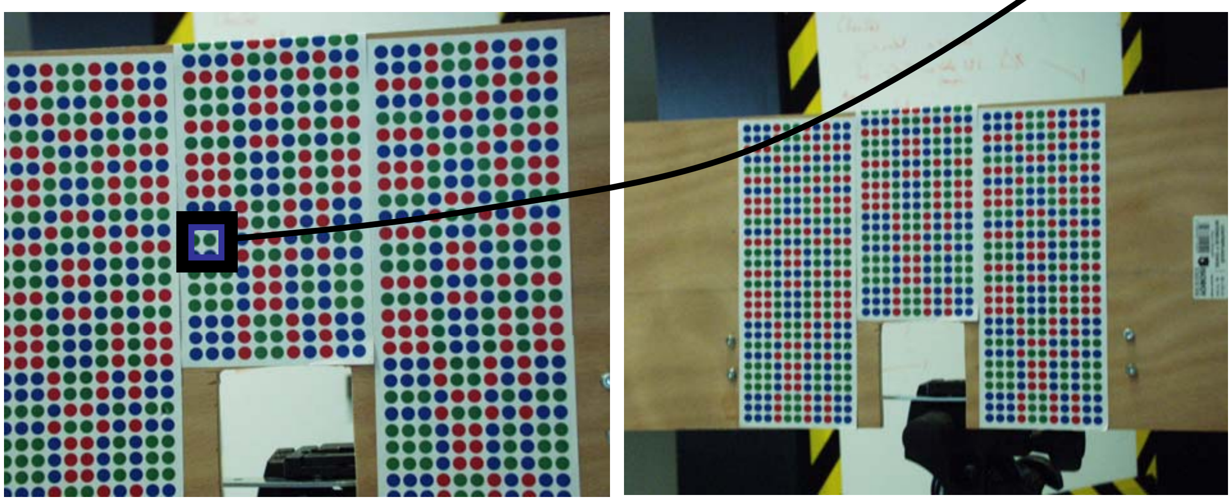
Non-parametric calibration

Basic idea

Approach using known motion:

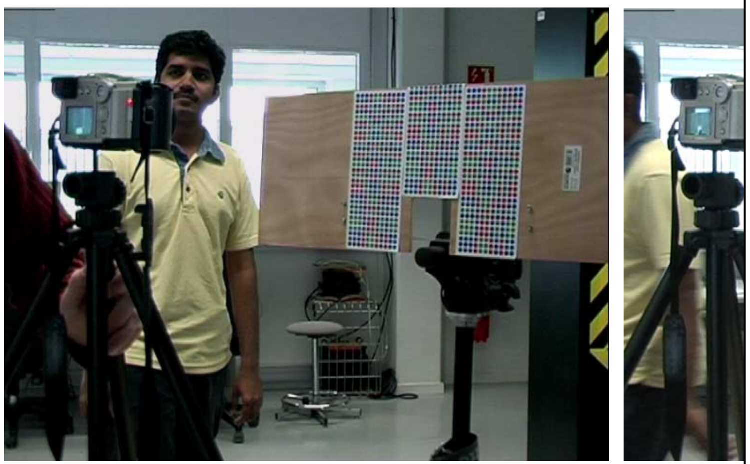


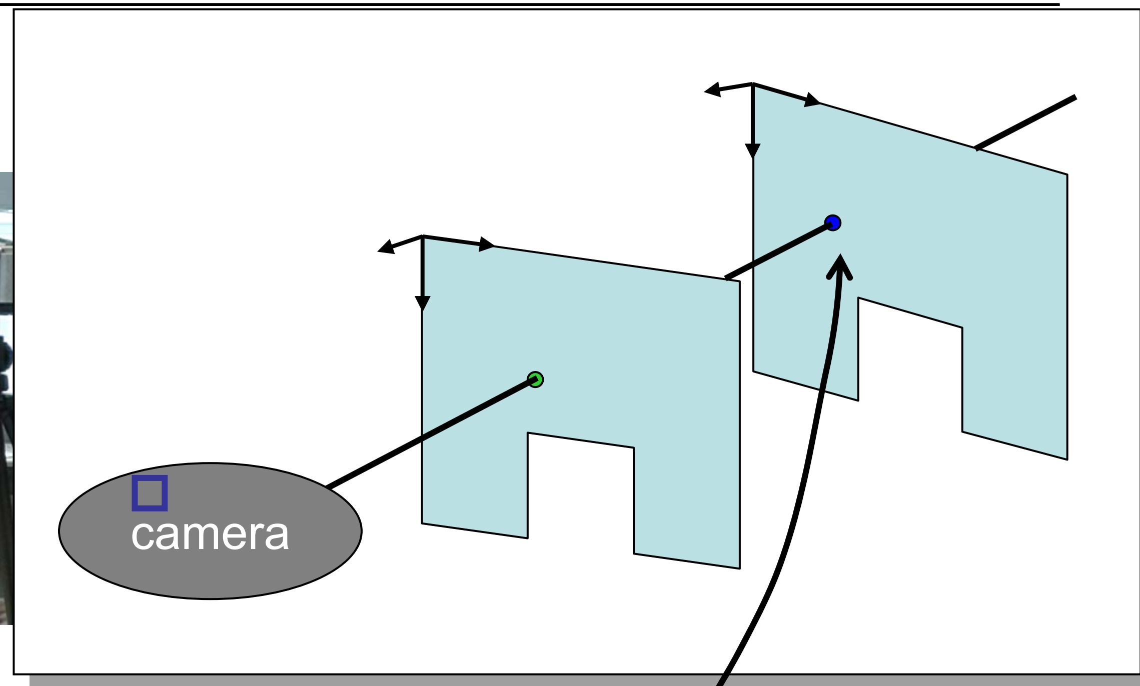


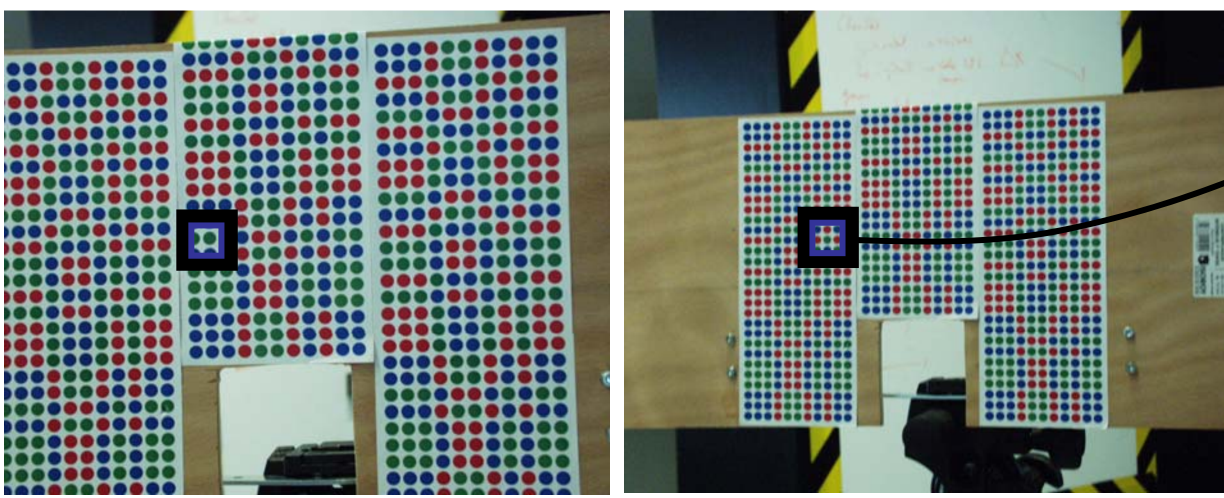


Non-parametric calibration

Approach using known motion:

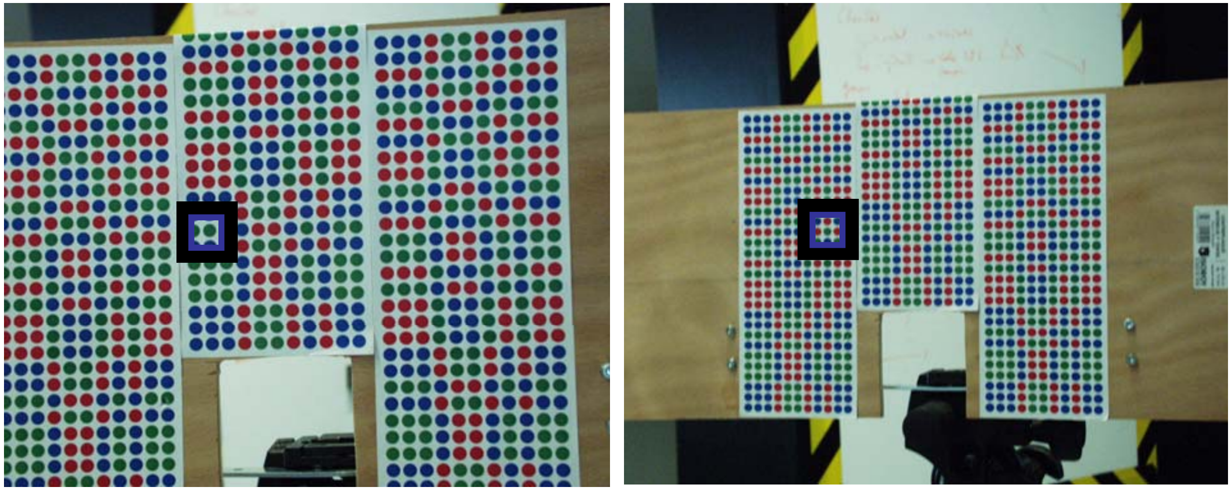






Non-parametric calibration — Matching

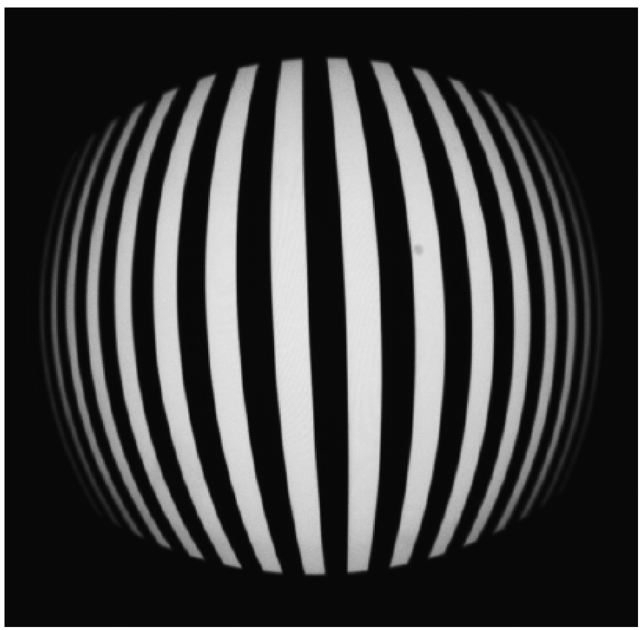
Using color coded grid:

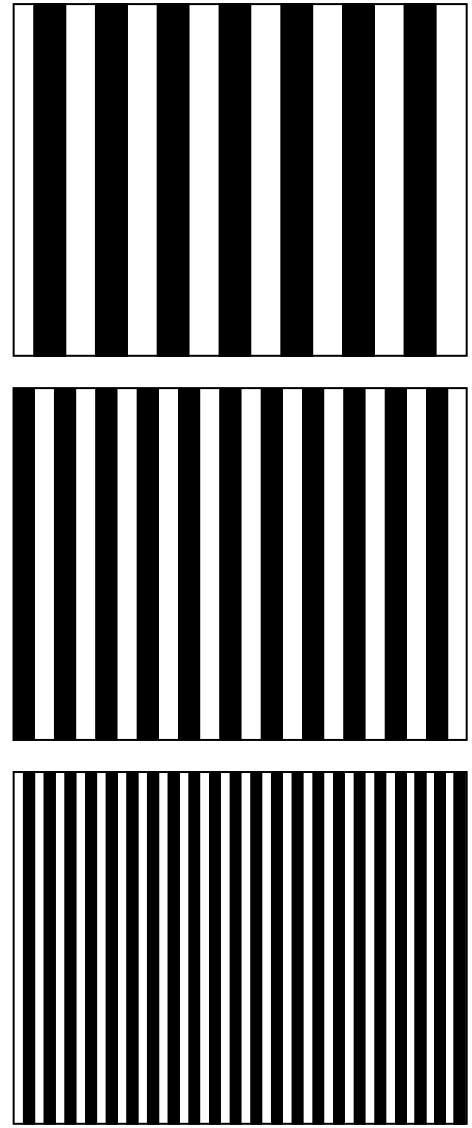


- Sparse matches, only for center pixels of circular targets
- We interpolate, for example using an homography:
  - for a pixel *p*, determine 4 closest pixels that have a match
  - compute 2D homography between these 4 image points and the matched points on the planar grid
  - apply this homography to compute point on grid that matches *p*

Better: structured light, e.g. acquiring images of a flat screen displaying a series of Gray code images (series of vertical and horizontal stripe patterns)

- Each screen pixel has its own unique sequence of black-white successions
- Dense matching between image and calibration grid (screen)

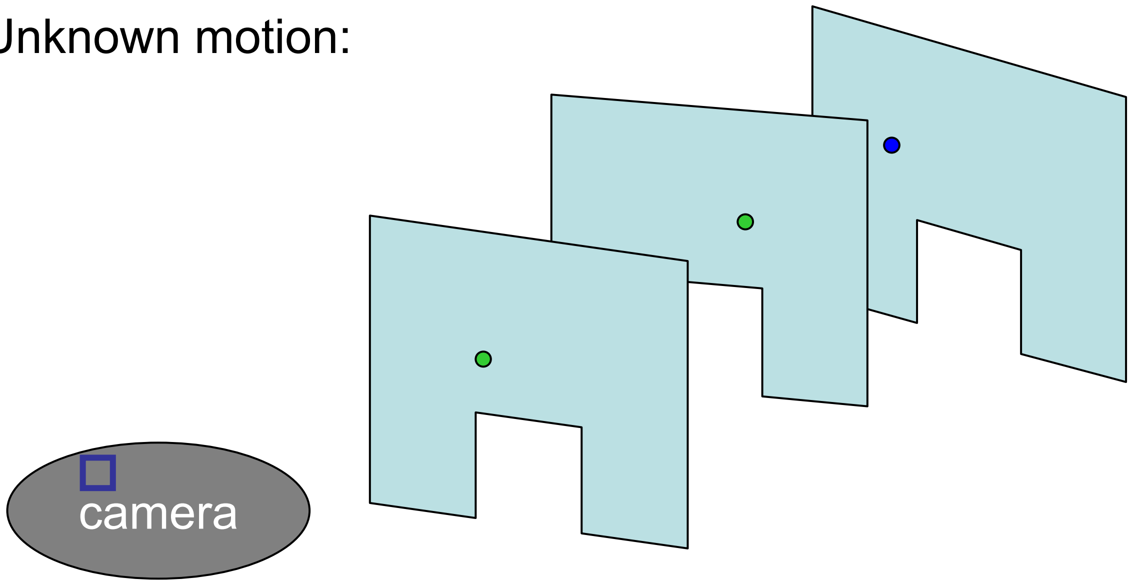




Non-parametric calibration — General approach

[Sturm-Ramalingam-ECCV'04]

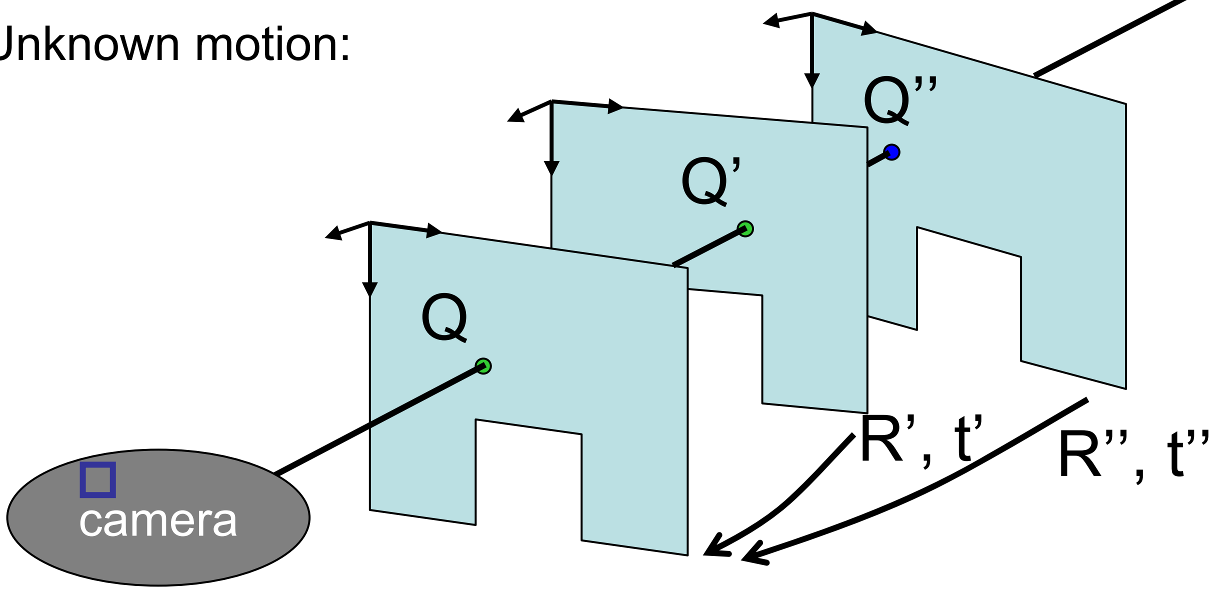
Unknown motion:



Non-parametric calibration General approach

[Sturm-Ramalingam-ECCV'04]

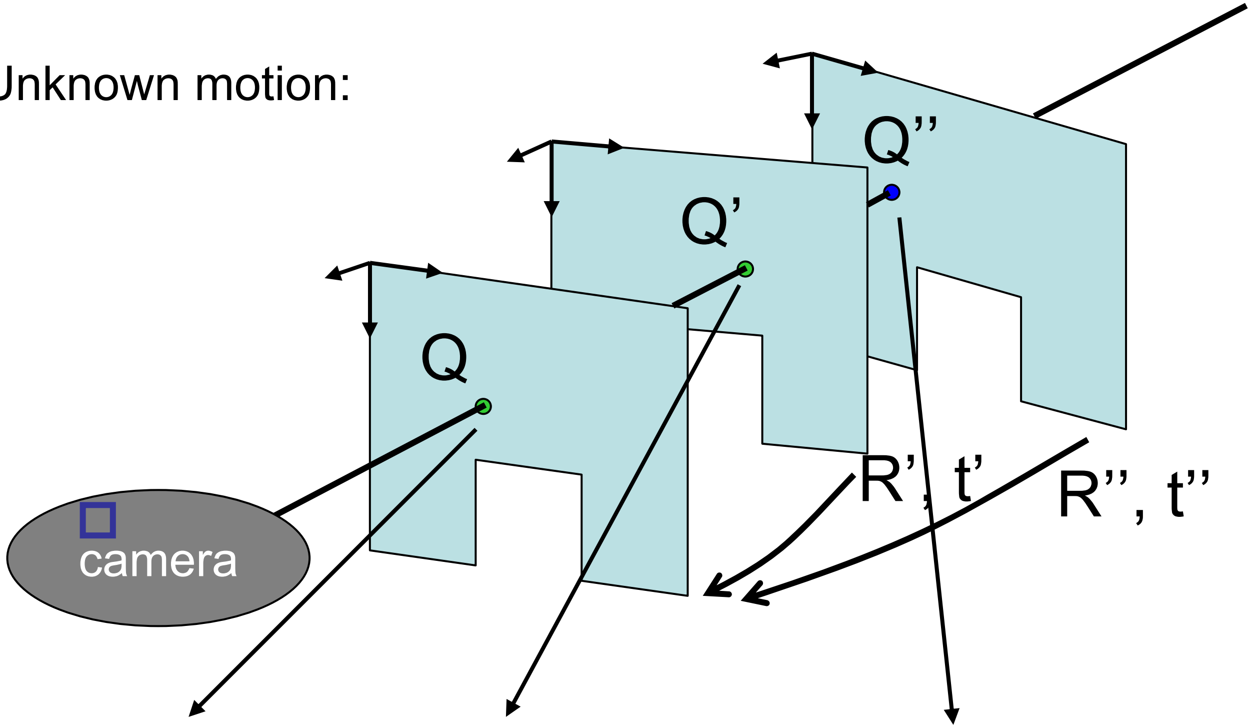
Unknown motion:



Estimate motions that make points collinear

[Sturm-Ramalingam-ECCV'04]

Unknown motion:



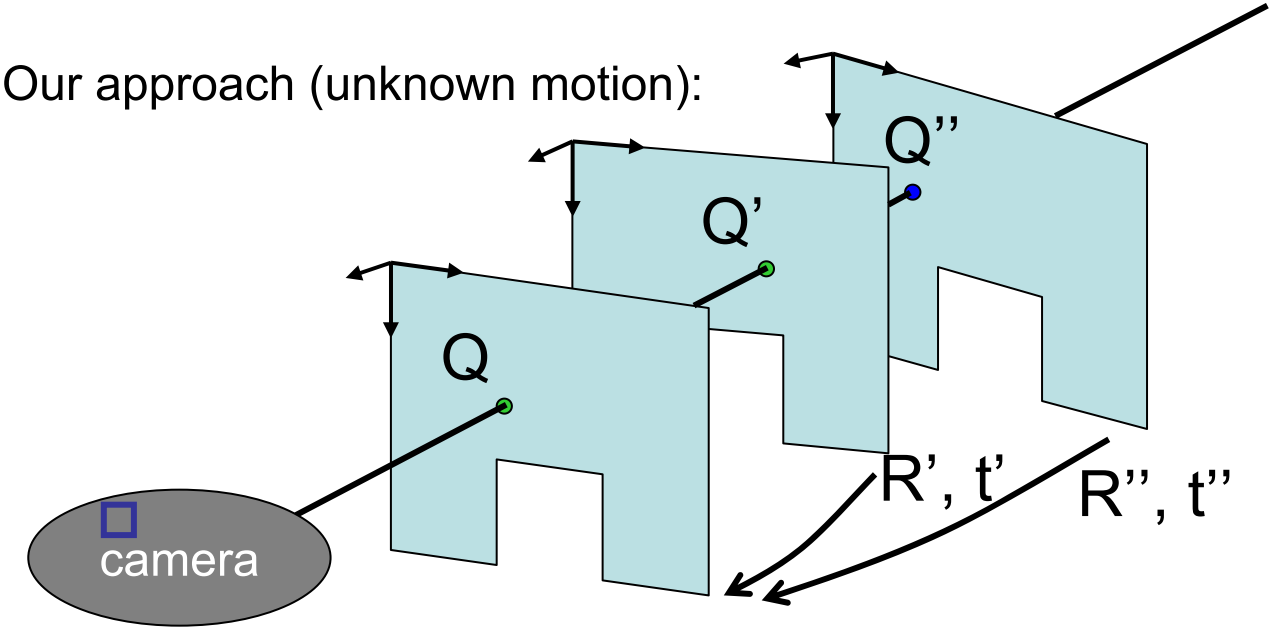
Estimate motions that make points collinear

$$\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{pmatrix} \quad \begin{pmatrix} R' \begin{pmatrix} Q'_1 \\ Q'_2 \\ Q'_3 \end{pmatrix} + t' Q'_4 \\ Q'_4 \end{pmatrix} \quad \begin{pmatrix} R'' \begin{pmatrix} Q''_1 \\ Q''_2 \\ Q''_3 \end{pmatrix} + t'' Q''_4 \\ Q''_4 \end{pmatrix}$$

Non-parametric calibration — General approach

[Sturm-Ramalingam-ECCV'04]

Our approach (unknown motion):



Estimate motions that make points collinear

$$\left(\begin{array}{ccc} Q_1 & & \\ Q_2 & R'\begin{bmatrix}Q'_1\\Q'_2\\Q'_3\end{bmatrix} + t'\,Q'_4 & R''\begin{bmatrix}Q''_1\\Q''_2\\Q''_3\end{bmatrix} + t''\,Q''_4 \\ Q_3 & & \\ Q_4 & Q'_4 & Q''_4 \end{array}\right)_{4\text{x}3} \longrightarrow \text{rank} < 3$$

$$\begin{pmatrix} Q_1 & R'\begin{bmatrix} Q'_1 \\ Q'_2 \\ Q'_3 \end{bmatrix} + t'\,Q'_4 & R''\begin{bmatrix} Q''_1 \\ Q''_2 \\ Q''_3 \end{bmatrix} + t''\,Q''_4 \\ Q_2 & & \\ Q_3 & & \\ Q_4 & Q'_4 & Q''_4 \end{pmatrix}_{4x3} \longrightarrow \text{rank} < 3$$

Non-parametric calibration — General approach

$$
\begin{pmatrix}
Q_1 & & \\
Q_2 & R' \begin{pmatrix} Q'_1 \\ Q'_2 \\ Q'_3 \end{pmatrix} + t' Q'_4 & R'' \begin{pmatrix} Q''_1 \\ Q''_2 \\ Q''_3 \end{pmatrix} + t'' Q''_4 \\
Q_3 & & \\
Q_4 & Q'_4 & Q''_4
\end{pmatrix}_{4x3} \longrightarrow \text{rank} < 3
$$

$$\Downarrow$$

$$
\begin{pmatrix}
Q_1 & & \\
Q_2 & R' \begin{pmatrix} Q'_1 \\ Q'_2 \\ Q'_3 \end{pmatrix} + t' Q'_4 & R'' \begin{pmatrix} Q''_1 \\ Q''_2 \\ Q''_3 \end{pmatrix} + t'' Q''_4 \\
Q_3 & & \\
\cancel{Q_4} & \cancel{Q'_4} & \cancel{Q''_4}
\end{pmatrix} \longrightarrow \det = 0
$$

$$\Downarrow$$

$$\det = \sum_{i,j,k=1}^{4} Q_i \, Q'_j \, Q''_k \, T_{i,j,k} = 0$$

a trifocal tensor

Non-parametric calibration — General approach

$$\begin{pmatrix} Q_1 & \multicolumn{1}{c}{\multirow{3}{*}{$R' \begin{bmatrix} Q'_1 \\ Q'_2 \\ Q'_3 \end{bmatrix} + t' Q'_4$}} & \multirow{3}{*}{$R'' \begin{bmatrix} Q''_1 \\ Q''_2 \\ Q''_3 \end{bmatrix} + t'' Q''_4$} \\ Q_2 & & \\ Q_3 & & \\ Q_4 & Q'_4 & Q''_4 \end{pmatrix}_{4\text{x}3} \longrightarrow \text{rank} < 3$$

$$\Downarrow$$

$$\begin{pmatrix} Q_1 & \multirow{3}{*}{$R' \begin{bmatrix} Q'_1 \\ Q'_2 \\ Q'_3 \end{bmatrix} + t' Q'_4$} & \multirow{3}{*}{$R'' \begin{bmatrix} Q''_1 \\ Q''_2 \\ Q''_3 \end{bmatrix} + t'' Q''_4$} \\ Q_2 & & \\ Q_3 & & \\ \cancel{Q_4} & \cancel{Q'_4} & \cancel{Q''_4} \end{pmatrix} \longrightarrow \det = 0$$

$$\Downarrow$$

$$\det = \sum_{i,j,k=1}^{4} Q_i \, Q'_j \, Q''_k \, T_{i,j,k} = 0$$

a trifocal tensor

4 such tensors exist, striking out one row in turn:

$$\begin{pmatrix} Q_1 & R' \begin{bmatrix} Q'_1 \\ Q'_2 \\ Q'_3 \end{bmatrix} + t' Q'_4 & R'' \begin{bmatrix} Q''_1 \\ Q''_2 \\ Q''_3 \end{bmatrix} + t'' Q''_4 \\ Q_2 & & \\ Q_3 & & \\ \cancel{Q_4} & \cancel{Q'_4} & \cancel{Q''_4} \end{pmatrix}_{4\times 3} \longrightarrow \det = 0$$

Each one has a particular structure, see the following slide for two examples

i	C_i	V_i	W_i
1	$Q_1Q'_1Q''_4$	0	R'_{31}
2	$Q_1Q'_2Q''_4$	0	R'_{32}
3	$Q_1Q'_3Q''_4$	0	R'_{33}
4	$Q_1Q'_4Q''_1$	0	$-R''_{31}$
5	$Q_1Q'_4Q''_2$	0	$-R''_{32}$
6	$Q_1Q'_4Q''_3$	0	$-R''_{33}$
7	$Q_1Q'_4Q''_4$	0	$t'_3 - t''_3$
8	$Q_2Q'_1Q''_4$	R'_{31}	0
9	$Q_2Q'_2Q''_4$	R'_{32}	0
10	$Q_2Q'_3Q''_4$	R'_{33}	0
11	$Q_2Q'_4Q''_1$	$-R''_{31}$	0
12	$Q_2Q'_4Q''_2$	$-R''_{32}$	0
13	$Q_2Q'_4Q''_3$	$-R''_{33}$	0
14	$Q_2Q'_4Q''_4$	$t'_3 - t''_3$	0
15	$Q_3Q'_1Q''_4$	$-R'_{21}$	$-R'_{11}$
16	$Q_3Q'_2Q''_4$	$-R'_{22}$	$-R'_{12}$
17	$Q_3Q'_3Q''_4$	$-R'_{23}$	$-R'_{13}$
18	$Q_3Q'_4Q''_1$	R''_{21}	R''_{11}
19	$Q_3Q'_4Q''_2$	R''_{22}	R''_{12}

i	C_i	V_i	W_i
20	$Q_3Q'_4Q''_3$	R''_{23}	R''_{13}
21	$Q_3Q'_4Q''_4$	$t''_2 - t'_2$	$t''_1 - t'_1$
22	$Q_4Q'_1Q''_1$	$R'_{21}R''_{31} - R''_{21}R'_{31}$	$R'_{11}R''_{31} - R''_{11}R'_{31}$
23	$Q_4Q'_1Q''_2$	$R'_{21}R''_{32} - R''_{22}R'_{31}$	$R'_{11}R''_{32} - R''_{12}R'_{31}$
24	$Q_4Q'_1Q''_3$	$R'_{21}R''_{33} - R''_{23}R'_{31}$	$R'_{11}R''_{33} - R''_{13}R'_{31}$
25	$Q_4Q'_1Q''_4$	$R'_{21}t''_3 - R'_{31}t''_2$	$R'_{11}t''_3 - R'_{31}t''_1$
26	$Q_4Q'_2Q''_1$	$R'_{22}R''_{31} - R''_{21}R'_{32}$	$R'_{12}R''_{31} - R''_{11}R'_{32}$
27	$Q_4Q'_2Q''_2$	$R'_{22}R''_{32} - R''_{22}R'_{32}$	$R'_{12}R''_{32} - R''_{12}R'_{32}$
28	$Q_4Q'_2Q''_3$	$R'_{22}R''_{33} - R''_{23}R'_{32}$	$R'_{12}R''_{33} - R''_{13}R'_{32}$
29	$Q_4Q'_2Q''_4$	$R'_{22}t''_3 - R'_{32}t''_2$	$R'_{12}t''_3 - R_{32}t''_1$
30	$Q_4Q'_3Q''_1$	$R'_{23}R''_{31} - R''_{21}R'_{33}$	$R'_{13}R''_{31} - R''_{11}R'_{33}$
31	$Q_4Q'_3Q''_2$	$R'_{23}R''_{32} - R''_{22}R'_{33}$	$R'_{13}R''_{32} - R''_{12}R'_{33}$
32	$Q_4Q'_3Q''_3$	$R'_{23}R''_{33} - R''_{23}R'_{33}$	$R'_{13}R''_{33} - R''_{13}R'_{33}$
33	$Q_4Q'_3Q''_4$	$R'_{23}t''_3 - R'_{33}t''_2$	$R'_{13}t''_3 - R'_{33}t''_1$
34	$Q_4Q'_4Q''_1$	$R''_{31}t'_2 - R''_{21}t'_3$	$R''_{31}t'_1 - R''_{11}t'_3$
35	$Q_4Q'_4Q''_2$	$R''_{32}t'_2 - R''_{22}t'_3$	$R''_{32}t'_1 - R''_{12}t'_3$
36	$Q_4Q'_4Q''_3$	$R''_{33}t'_2 - R''_{23}t'_3$	$R''_{33}t'_1 - R''_{13}t'_3$
37	$Q_4Q'_4Q''_4$	$t'_2t''_3 - t'_3t''_2$	$t'_1t''_3 - t''_1t'_3$

Calibration algorithm:

(1) Take images of calibration object in different poses

(2) 2D-3D matching (pixels to points on object)

(3) Estimation of tensors, based on linear equations

$$\sum_{i,j,k=1}^{4} Q_i \, Q'_j \, Q''_k \, T_{i,j,k} = 0$$

and taking into account the tensors' structure (e.g. coefficients that are zero)

(4) Extraction of motion parameters from tensors:

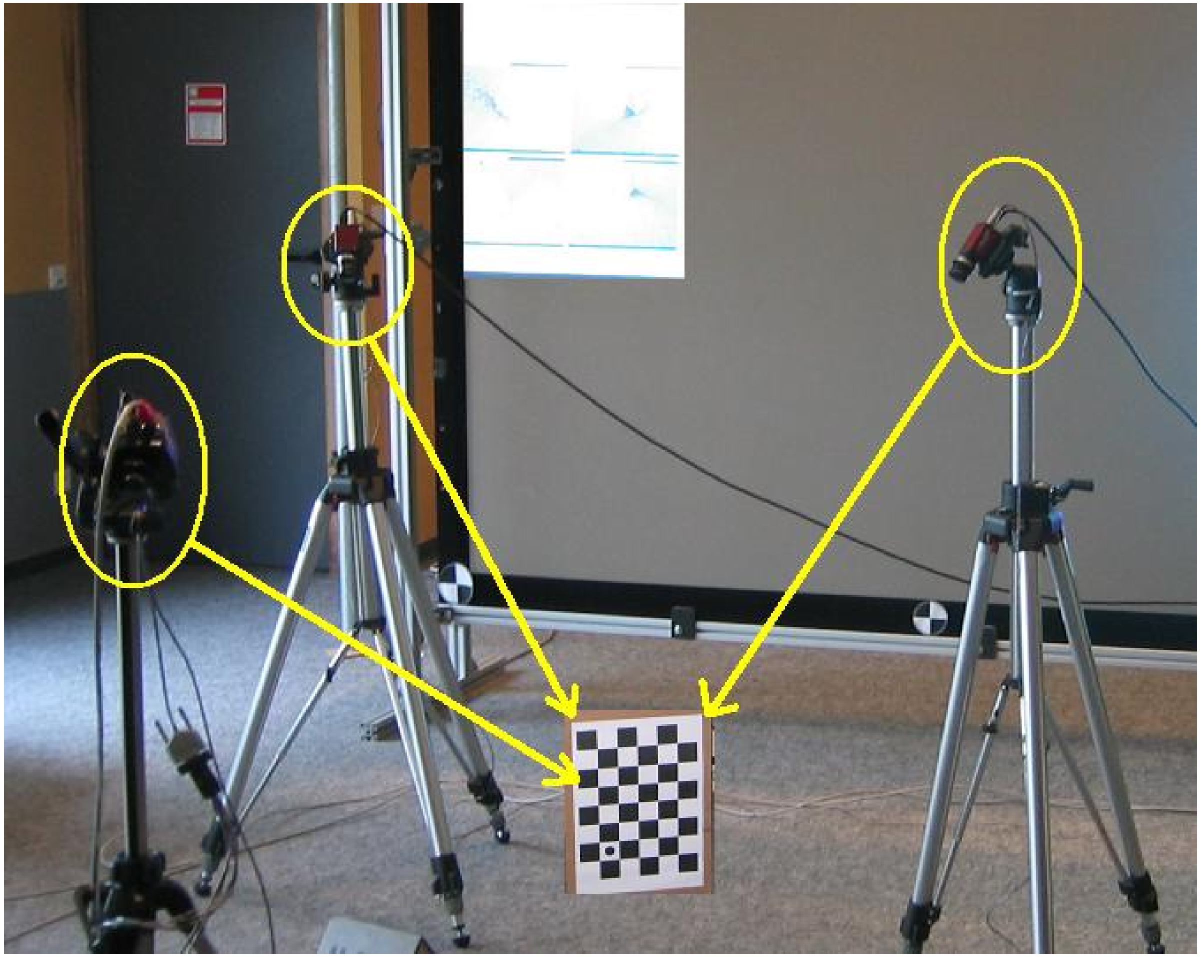
- some can be directly read off (some rotation coefficients, cf. previous slide)
- others can be computed using orthonormality constraints on R' and R''

(5) Put calibration grids in same 3D coordinate system

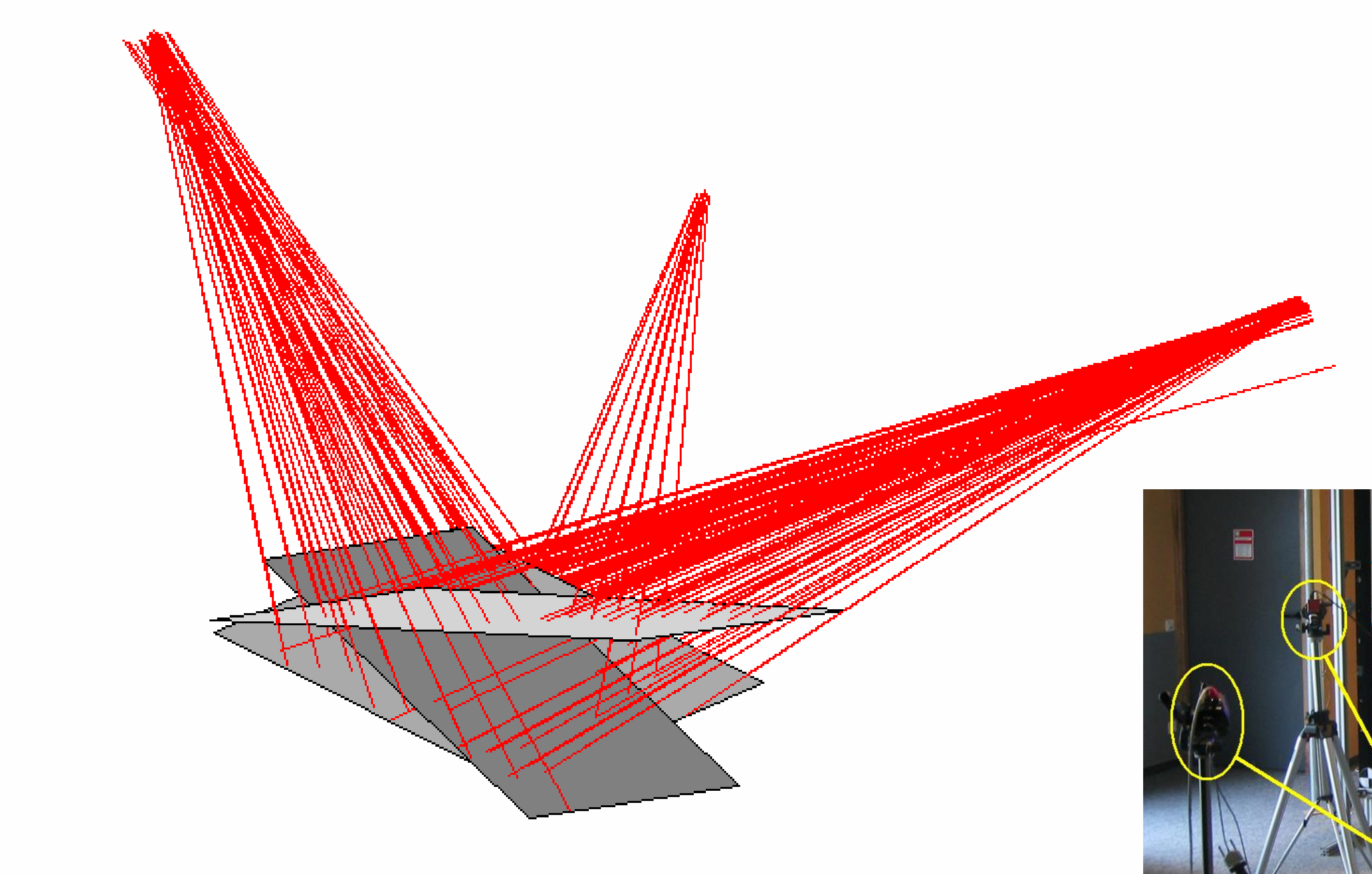
(6) Compute projection rays: for each pixel join the associated calibration points

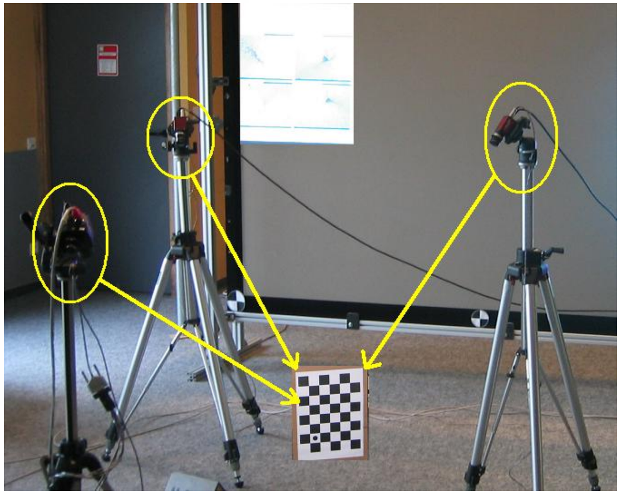
(7) Bundle adjustment

Results for non-central camera
(multi-camera system, considered as single non-central camera):

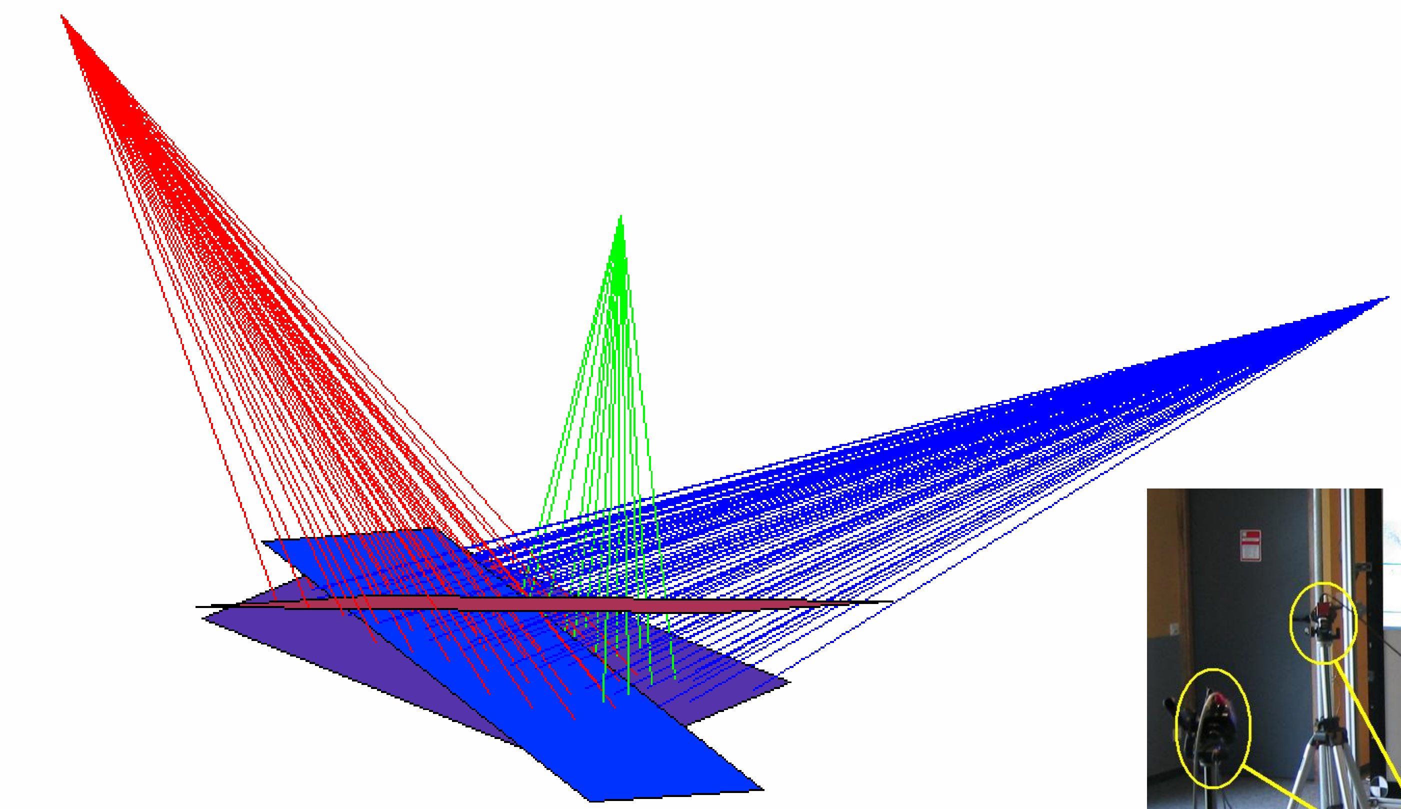


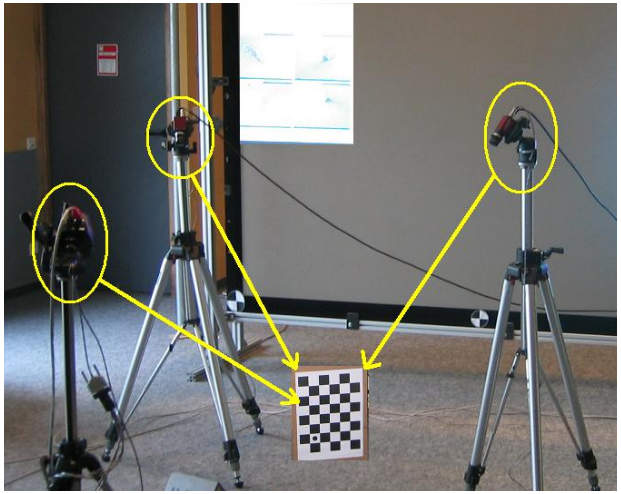
Results for non-central camera:





Results for non-central camera: after constraining rays into central clusters





Intermediate discussion:

- the approach is designed for 3D calibration objects

  → ***variant for using* planar *calibration objects (see next)***

- this approach uses *exactly* 3 images
  - only pixels covered by all 3 images of the calibration grid are calibrated
    → especially with large field of view, difficult to calibrate whole image
  - results may not be highly accurate

  → ***methods for using multiple images (see later)***

- the approach allows to calibrate non-central cameras!
- BUT: if used with images acquired by central camera
  - tensors are not computed uniquely (linear equation system of too low rank)
    → calibration fails

  → ***variant of the approach for central cameras and a few other special cases (see later)***

Using planar calibration grids: $Q_3 = Q'_3 = Q''_3 = 0$

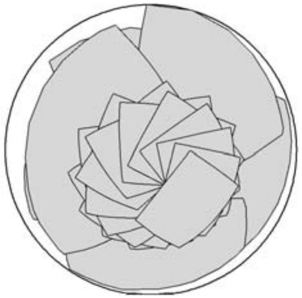
$$\begin{pmatrix} Q_1 & R'\begin{bmatrix} Q'_1 \\ Q'_2 \\ 0 \end{bmatrix} + t'Q'_4 & R''\begin{bmatrix} Q''_1 \\ Q''_2 \\ 0 \end{bmatrix} + t''Q''_4 \\ Q_2 & & \\ 0 & & \\ Q_4 & Q'_4 & Q''_4 \end{pmatrix}_{4\text{x}3} \longrightarrow \text{rank} < 3$$

- Tensors are different
- Extraction of motion parameters is more complicated, but possible

Non-parametric calibration — Using multiple images

Using multiple images:

…



- Idea:

  (1) Initial calibration using 3 images and above approach

  (2) Consider an additional image:

  - Compute pose of calibration grid using already available calibration information
  - Extend the calibration to pixels covered by the additional grid

  (3) Repeat (2) for all images. Then, bundle adjustment.

- Also:

  Possibility of performing initial calibration using multiple images if the regions covered by the grids mutually overlap

[Ramalingam-etal-CVPR'05]

Non-parametric calibration

If the calibration approach is used with images acquired by a central camera, then tensors are not computed uniquely (linear equation system of too low rank)

→ calibration fails

We thus consider a hierarchy of generic imaging models:

- Non-central
- Axial (non-central, but all projection rays touch a line, the **camera axis**)
  - linear push-broom camera
  - catadioptric system using a spherical mirror
- Central (all projection rays go through a single point, the **optical center**)

Calibration approaches for these three models have been developed

Approach for central model:

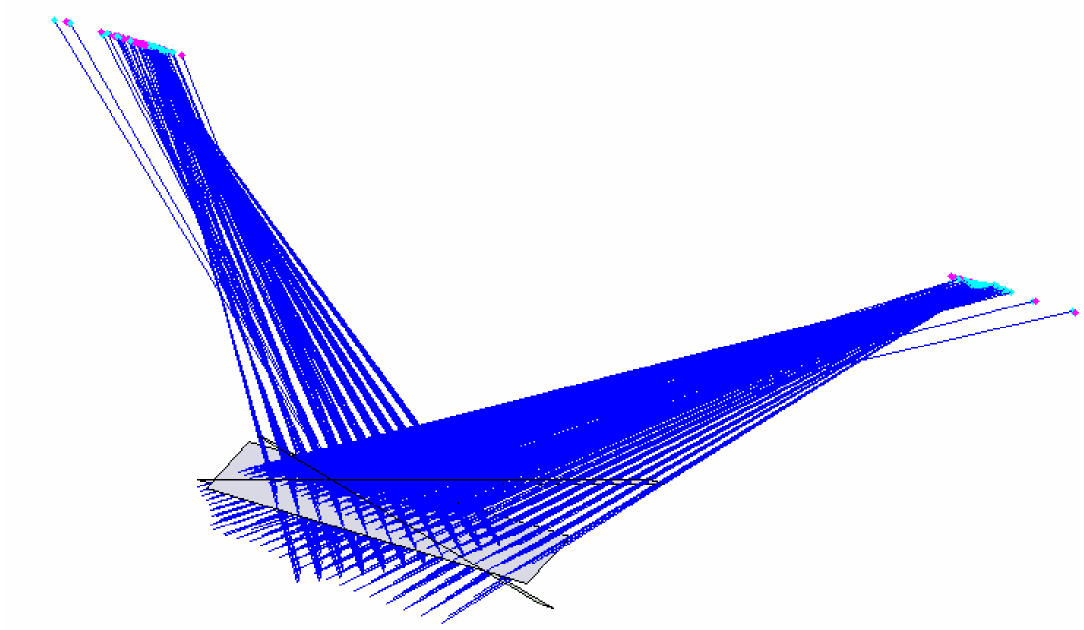
- Two images are sufficient (if 3D calibration object)
- Introduce coordinates of optical center C as unknowns
- Constraint: collinearity of optical center and two calibration points

$$\begin{pmatrix} C_1 & Q_1 & \\ C_2 & Q_2 & R' \begin{bmatrix} Q'_1 \\ Q'_2 \\ Q'_3 \end{bmatrix} + t' Q'_4 \\ C_3 & Q_3 & \\ C_4 & Q_4 & Q'_4 \end{pmatrix}_{4\text{x}3} \longrightarrow \text{rank} < 3$$

- Gives rise to yet another set of tensors
- Extraction of motion parameters and optical center from tensors

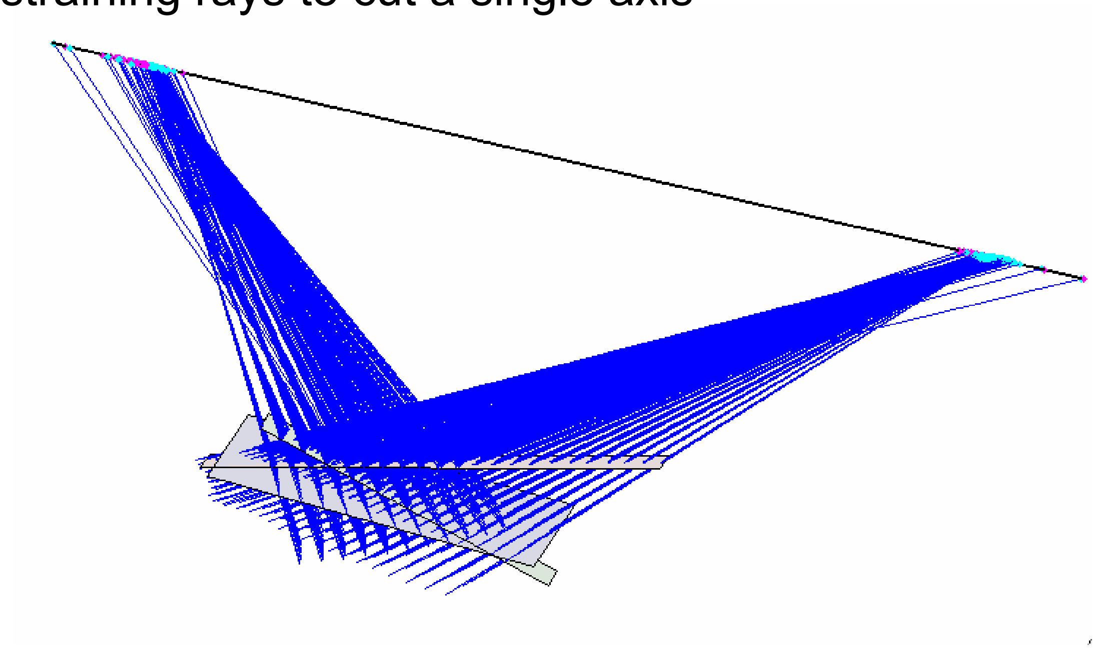
Similar approach for axial camera model (not shown here) [Ramalingam-etal-ACCV'06]

Results for axial camera model
(for a stereo system, considered as single axial camera):



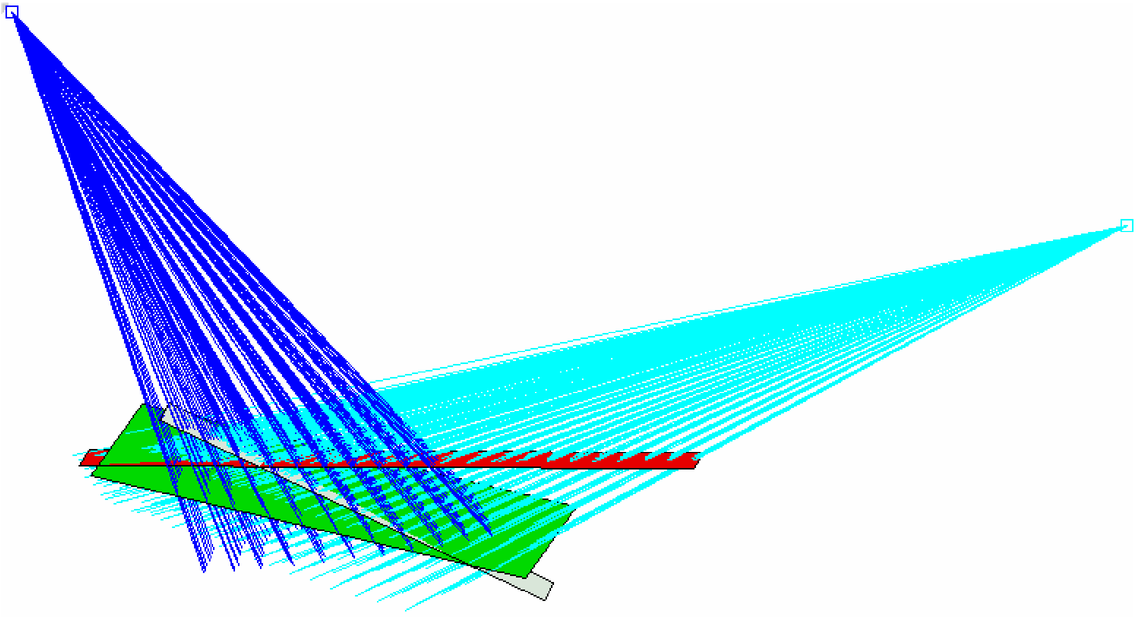
Results for axial camera model
(for a stereo system, considered as single axial camera):

After constraining rays to cut a single axis



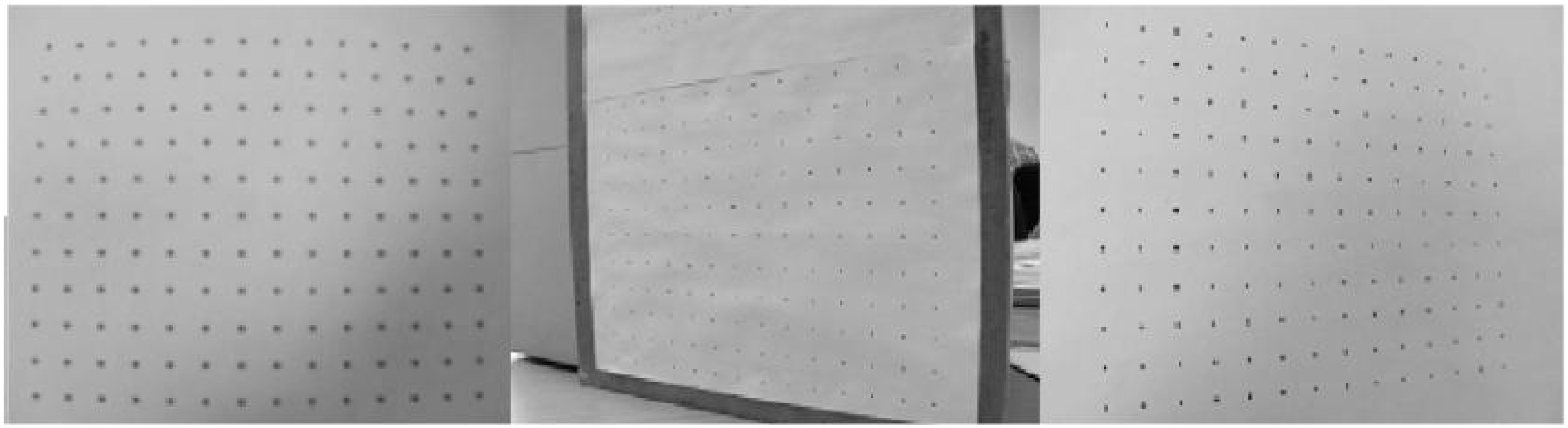
Results for axial camera model
(for a stereo system, considered as single axial camera):

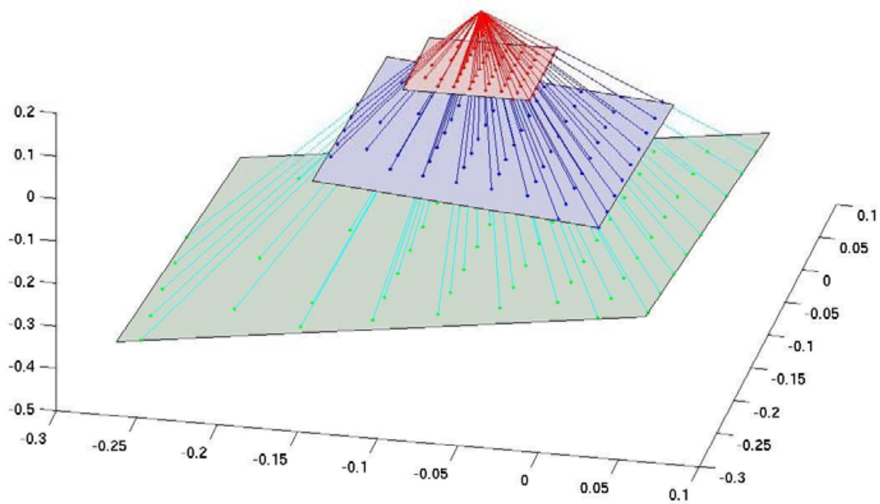
After constraining rays into central clusters

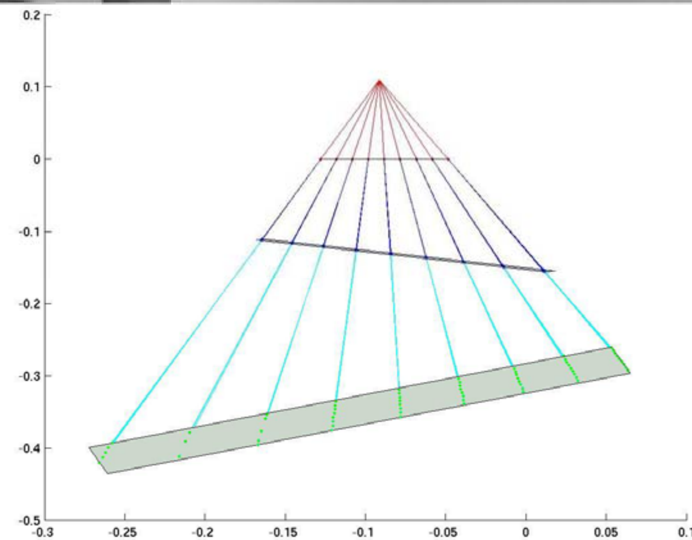


Non-parametric calibration Approach for central model

Central model applied on pinhole camera with slight radial distortion

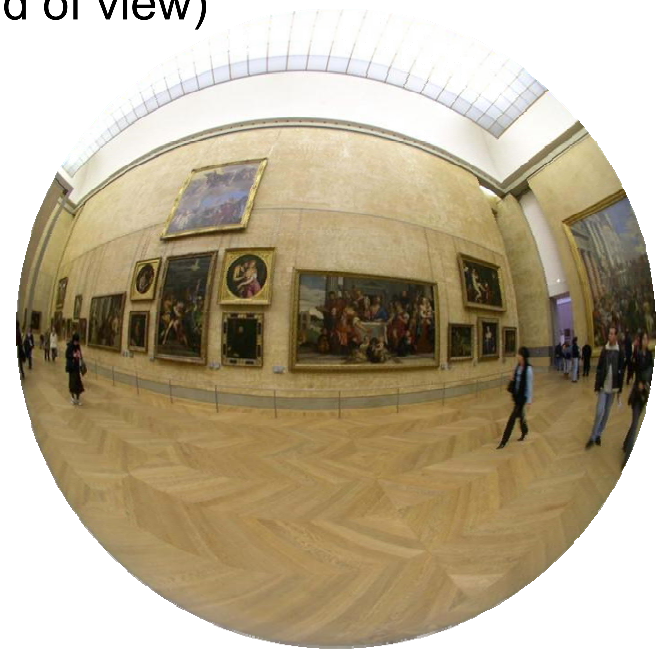






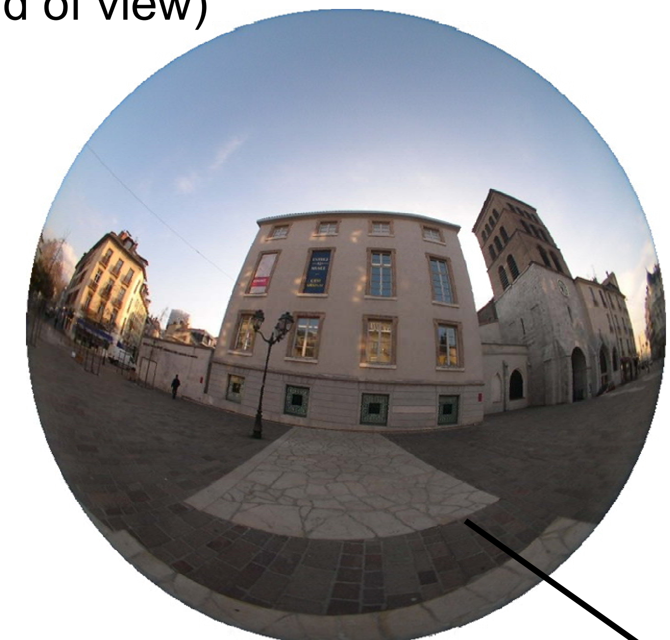
Non-parametric calibration   Approach for central model

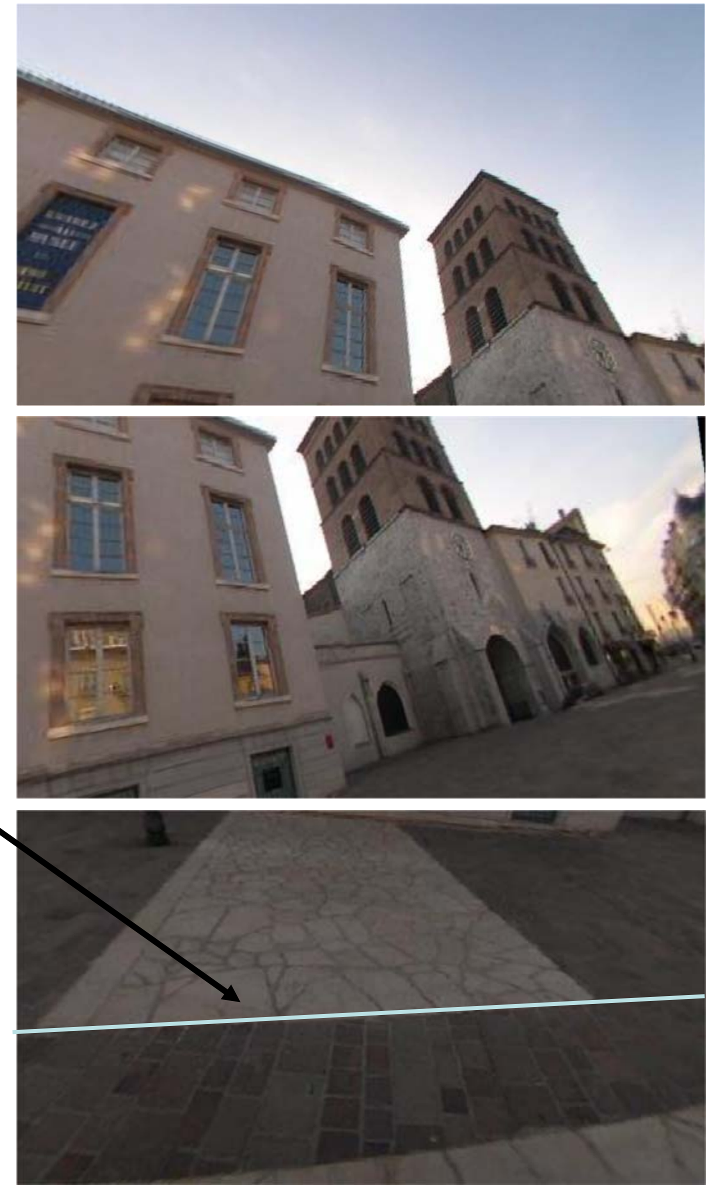
Results for fisheye camera
(183° field of view)



Non-parametric calibration   Approach for central model

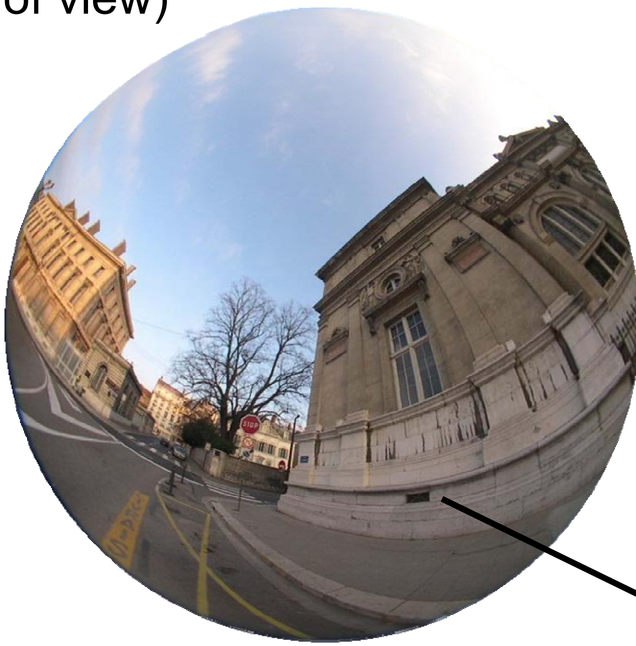
Results for fisheye camera
(183° field of view)

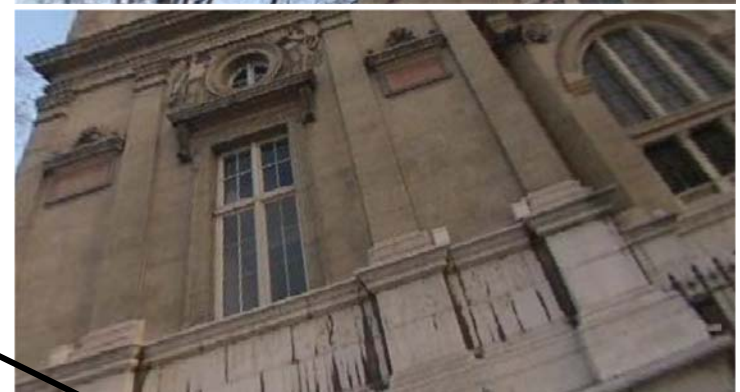




Non-parametric calibration   Approach for central model

Results for fisheye camera
(183° field of view)





Distortion correction:

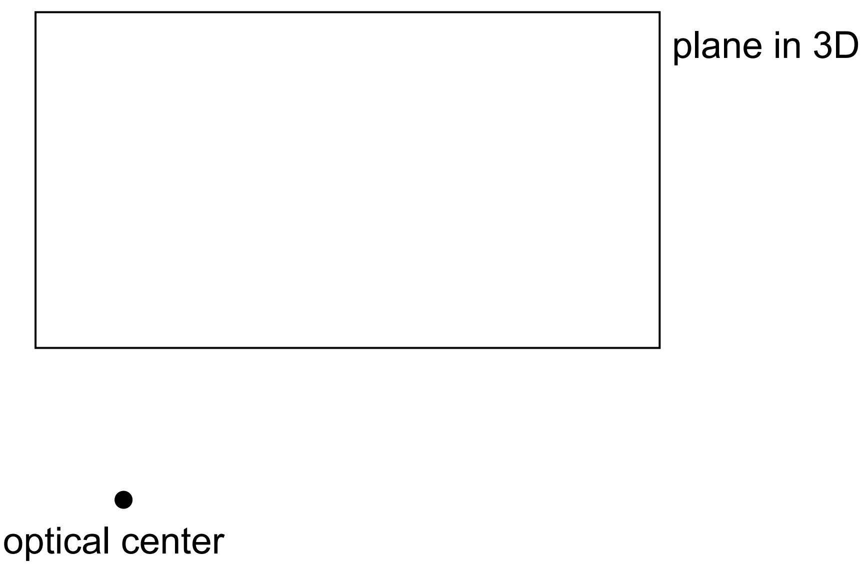
- Classical approach is based on analytical relationship between distorted and undistorted image coordinates (based on parametric calibration model)

- Generalization: approach for non-parametric calibration

Non-parametric calibration — Distortion correction

General distortion correction approach (for central cameras):

- Input: image and calibration information (projection rays for all pixels)
- Idea:
  - attribute pixels' color to their projection rays in 3D
  - define some plane in 3D
  - cut all projection rays: at each intersection point, paint a dot of the ray's color
  - the painted plane shows a distortion corrected image



General distortion correction approach (for central cameras):

- Input: image and calibration information (projection rays for all pixels)
- Idea:
  - attribute pixels' color to their projection rays in 3D
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  - cut all projection rays: at each intersection point, paint a dot of the ray's color
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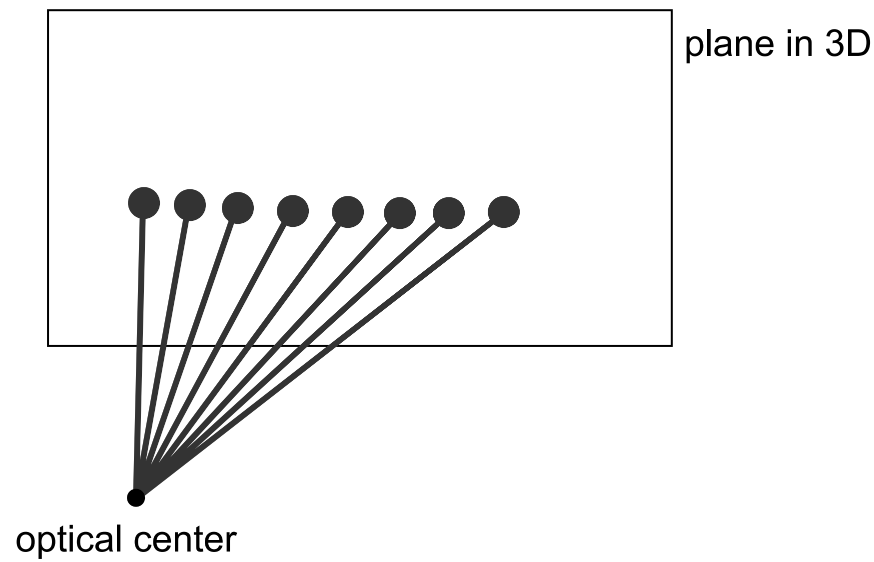

General distortion correction approach (for central cameras):

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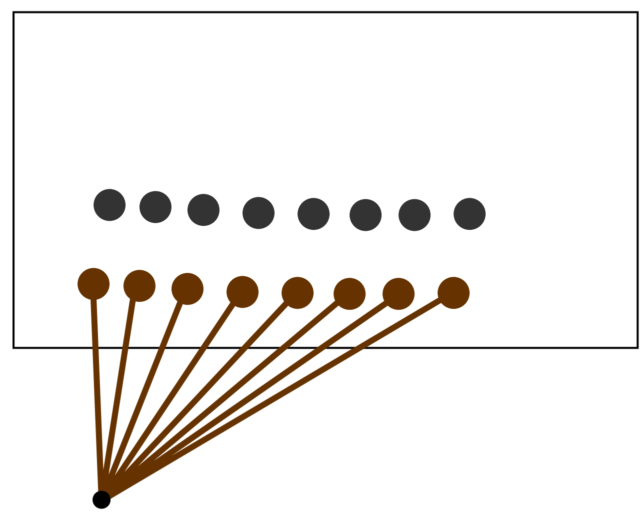

Non-parametric calibration — Distortion correction

General distortion correction approach (for central cameras):

- Input: image and calibration information (projection rays for all pixels)
- Idea:
  - attribute pixels' color to their projection rays in 3D
  - define some plane in 3D
  - cut all projection rays: at each intersection point, paint a dot of the ray's color
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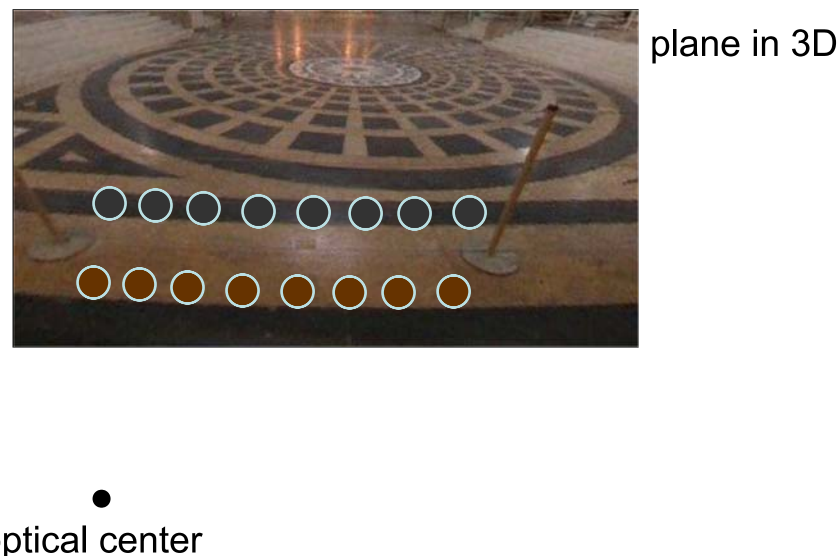


Non-parametric calibration — Discussion

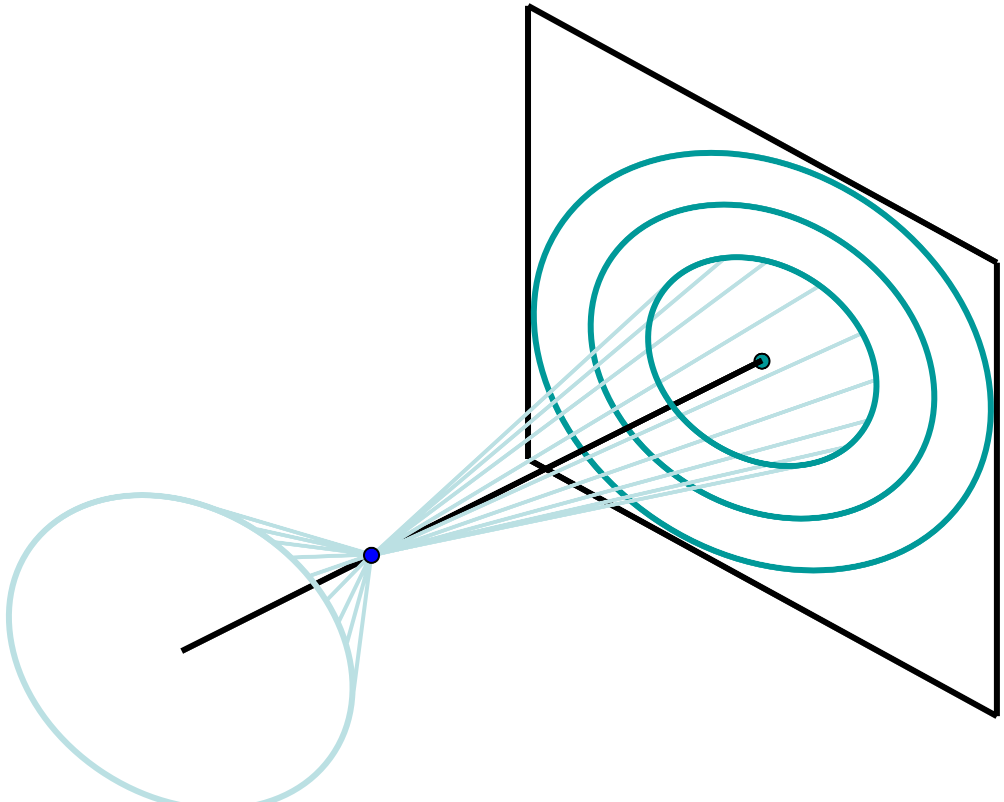
- General approach that allows to calibrate any camera
- Variants for central and axial camera modes
- Variants for using planar or 3D calibration objects

- How about stability?
  - Possible overfitting when calibrating “not very non-central cameras” with the general approach (result may be worse than with the central approach)
  - Stability depends on:
    - amount of “non-centrality”
    - number of images
    - accuracy of matches
  - If unstable:
    use more images, regularization, assumption of radial symmetry, …

Non-parametric calibration — Discussion

- Here, pixel-wise discretization of camera model
- Any other discretization (sub-pixel or super-pixel) is possible
- Trade-off between
  - potential accuracy of calibration (the finer the discretization, the better)
  - potential instability (the finer the discretization, the more unknowns…)

Interesting special case: radially symmetric cameras



Calibration:

Computation of distortion center and

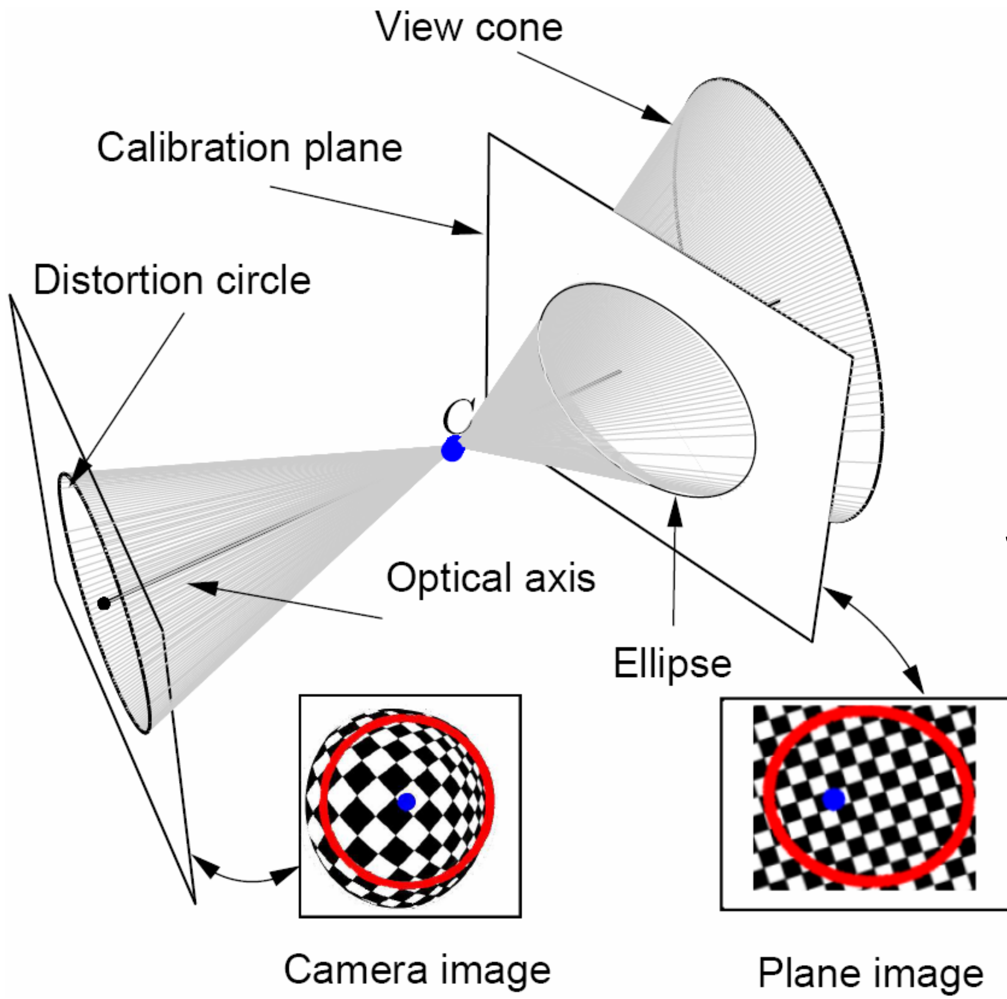
distortion function: *radius → view angle / focal length*

Note: each distortion circle ≡ perspective camera

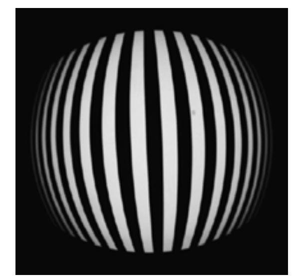
[Tardif-Sturm-OMNIVIS'05]

Calibration

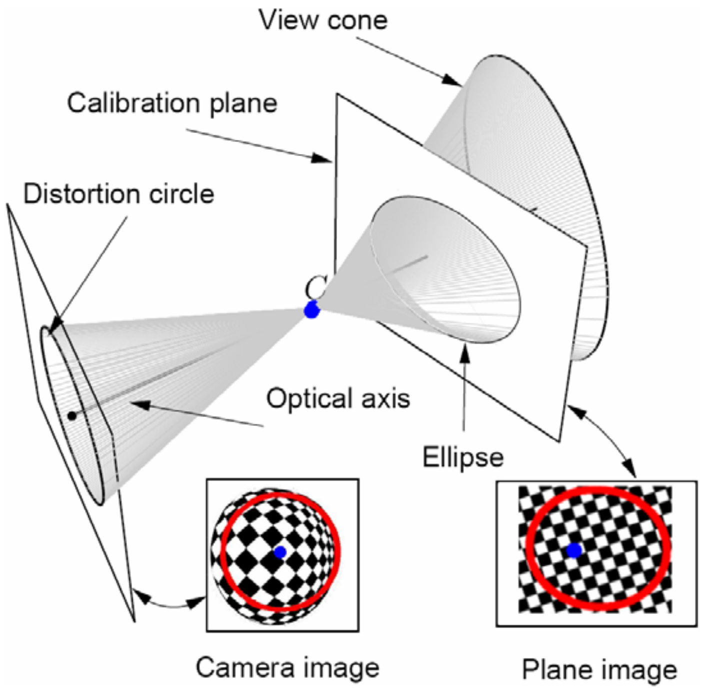
Camera



Screen



Calibration

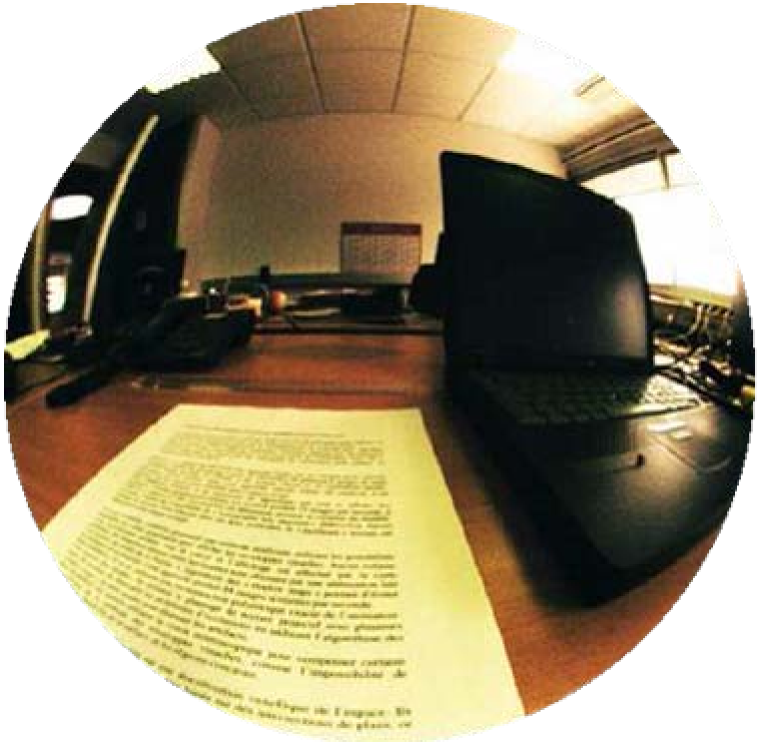


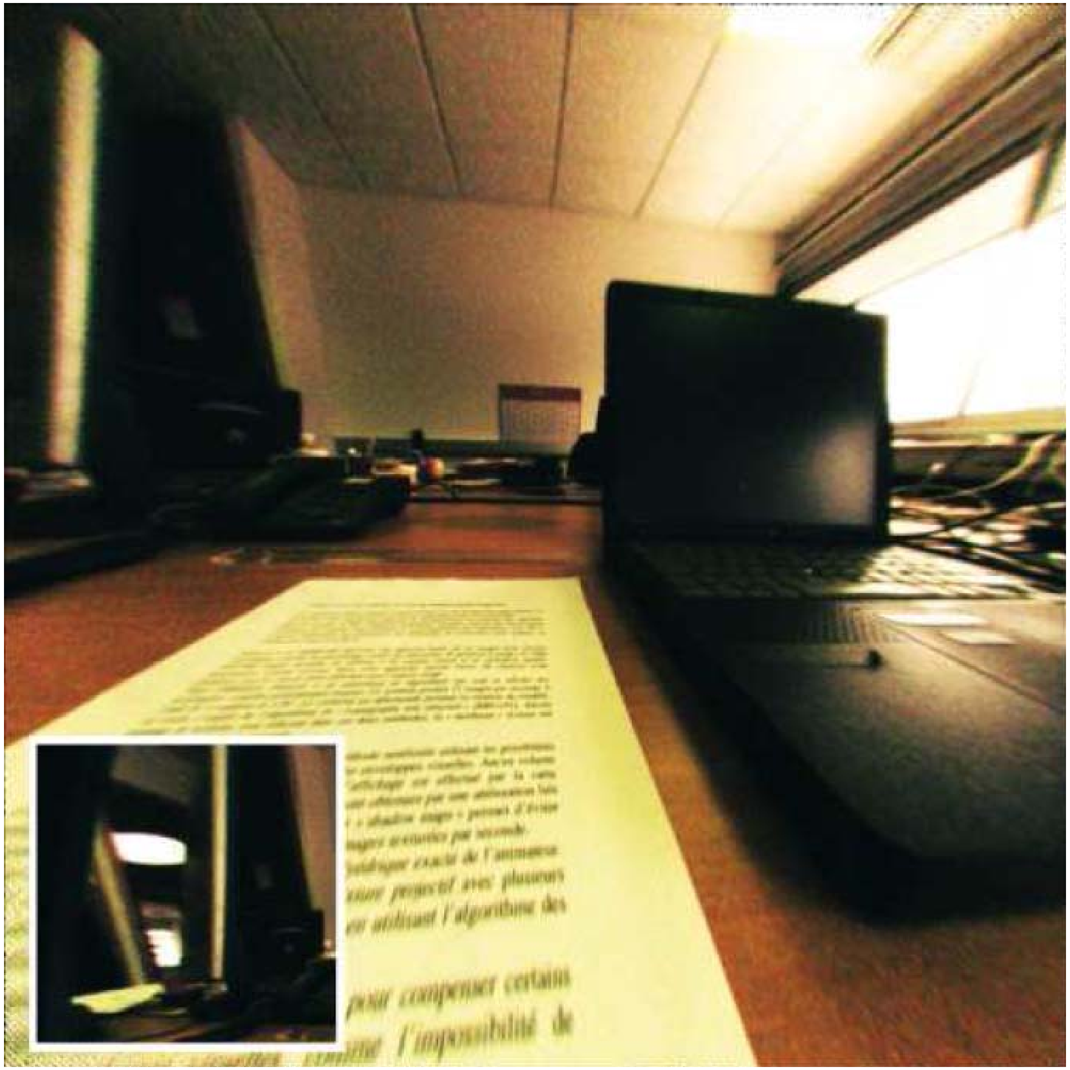
(1) For each distortion circle:
- compute homography screen ↔ image
- run classical plane-based calibration [Zhang'99, Sturm'99]

(2) Bundle adjustment over all distortion circles

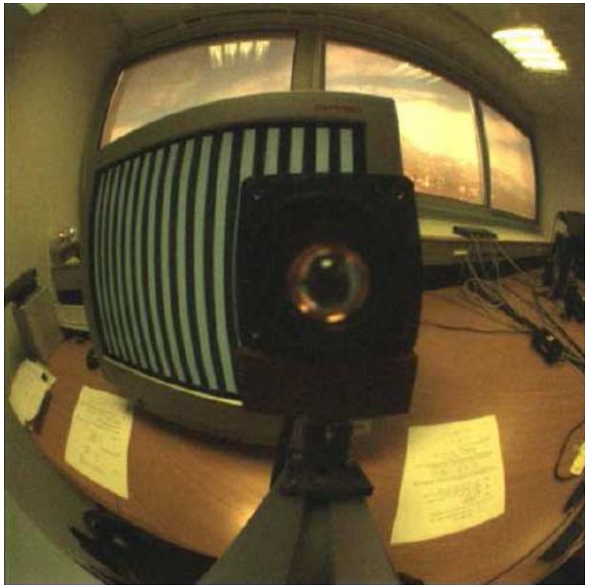
Non-parametric calibration Radially symmetric cameras

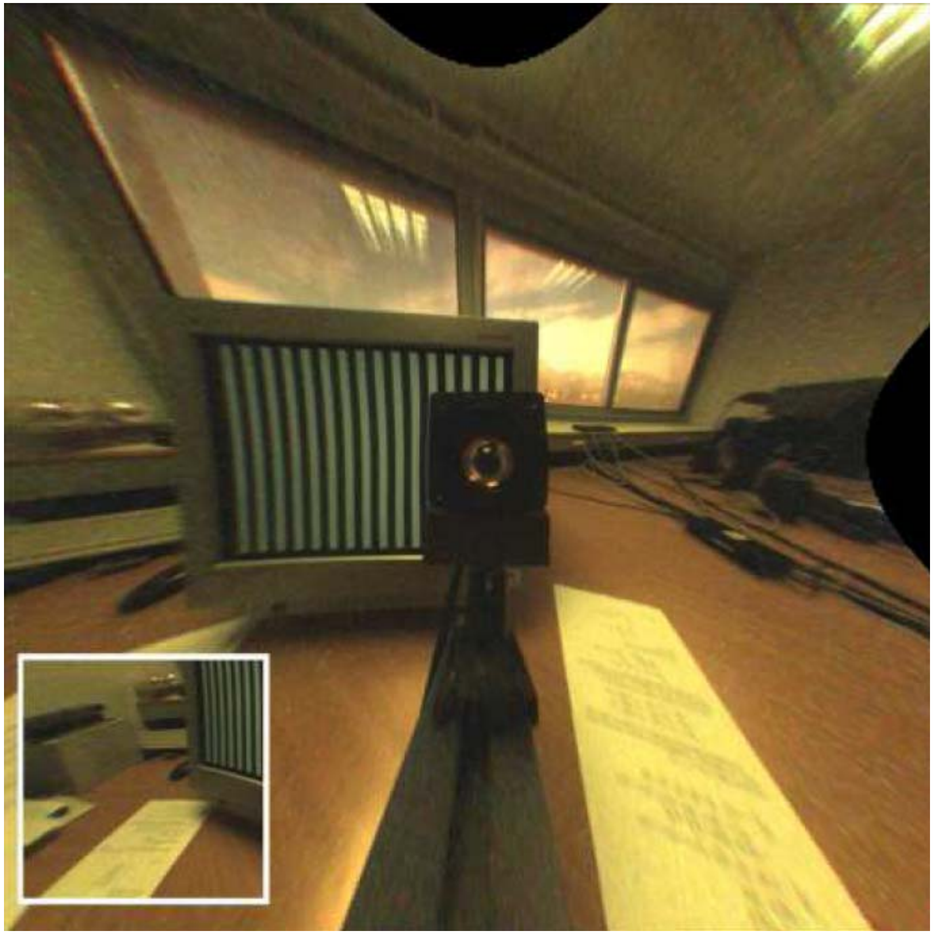
Result of distortion correction for fisheye





Result for homemade “Christmas camera”





Non-parametric calibration — Radially symmetric cameras

Discussion:

- Effective calibration approach for general radial distortion
- Handles field of view larger than 180° !
- Approaches for both, central and non-central cameras
- A single image is sufficient (but for stability, more images should be used)

Other recent work:

- Epipolar geometry of radially symmetric cameras [Barreto-Daniilidis-ICCV'05]
- Multi-view geometry and self-calibration of radial cameras [Thirthala-Pollefeys-ICCV'05]
- Self-calibration from two or more views of an arbitrary scene plane [Tardif-etal-ECCV'06]
- Direct method for computation of distortion center [Hartley-Kang-ICCV'05]

Contents

- Introduction
- General imaging models
- Non-parametric calibration and distortion correction
- Non-parametric self-calibration
- Structure-from-motion

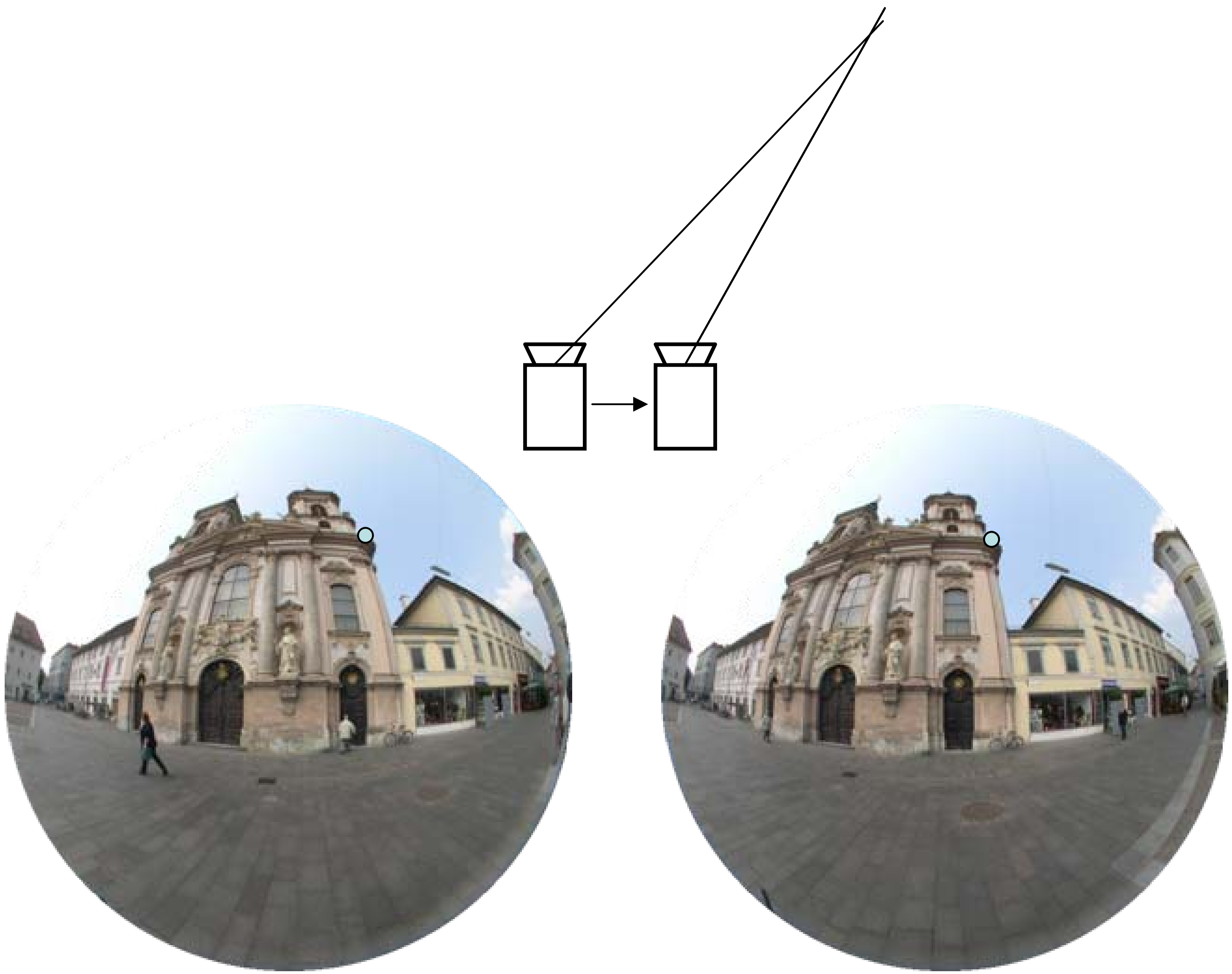
Non-parametric self-calibration

Self-calibration:

- The only existing works use special camera motions and only work for central cameras
  [Ramalingam-etal-OMNIVIS'05,Nistér-etal-ICCV'05,Grossman-etal-CVPR'06]
- In the following: illustration of basic idea
- Goal: compute directions of projection rays
- Input:
  - images taken under special camera motions
  - point tracks

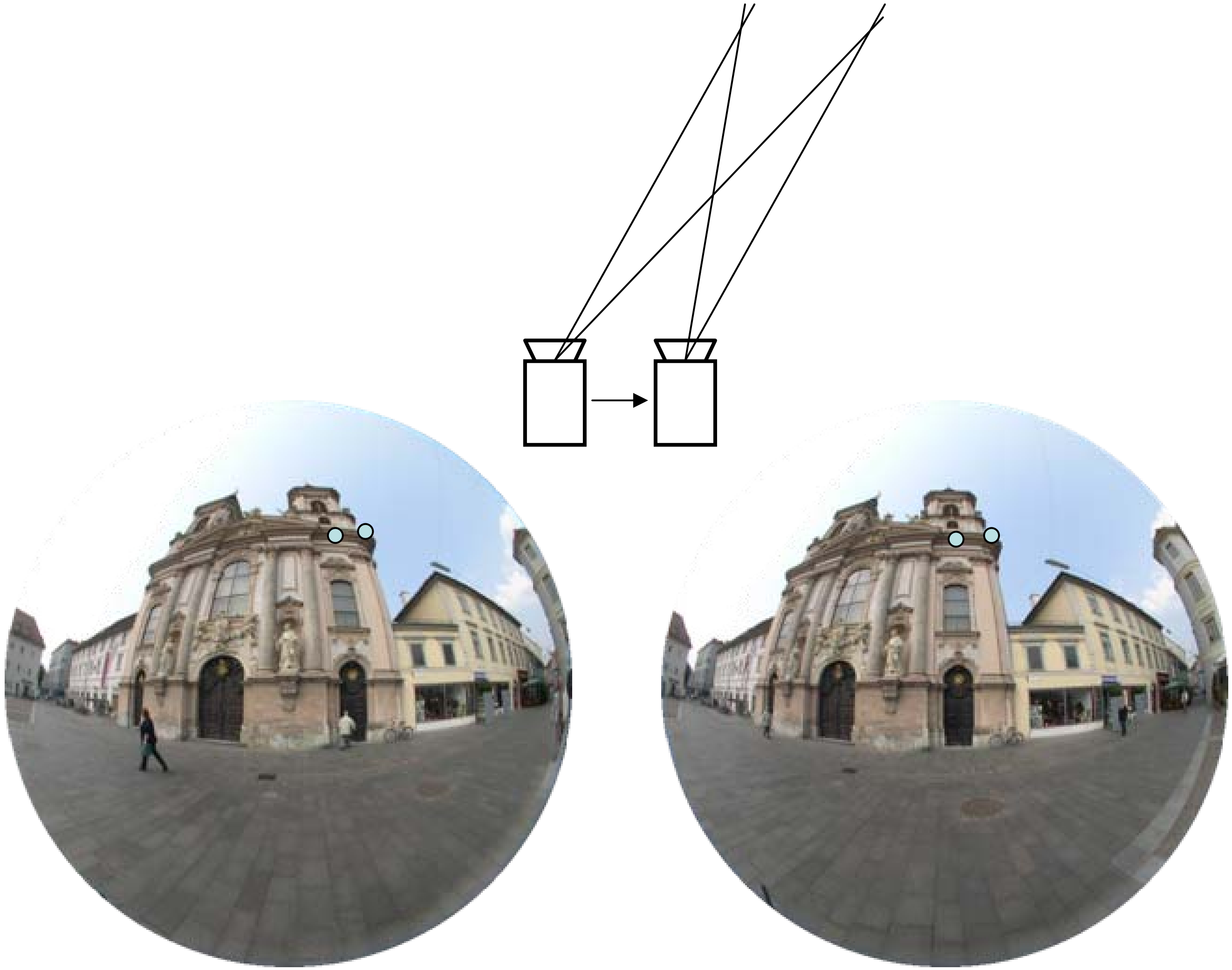
Non-parametric self-calibration

Flow curves for pure translations



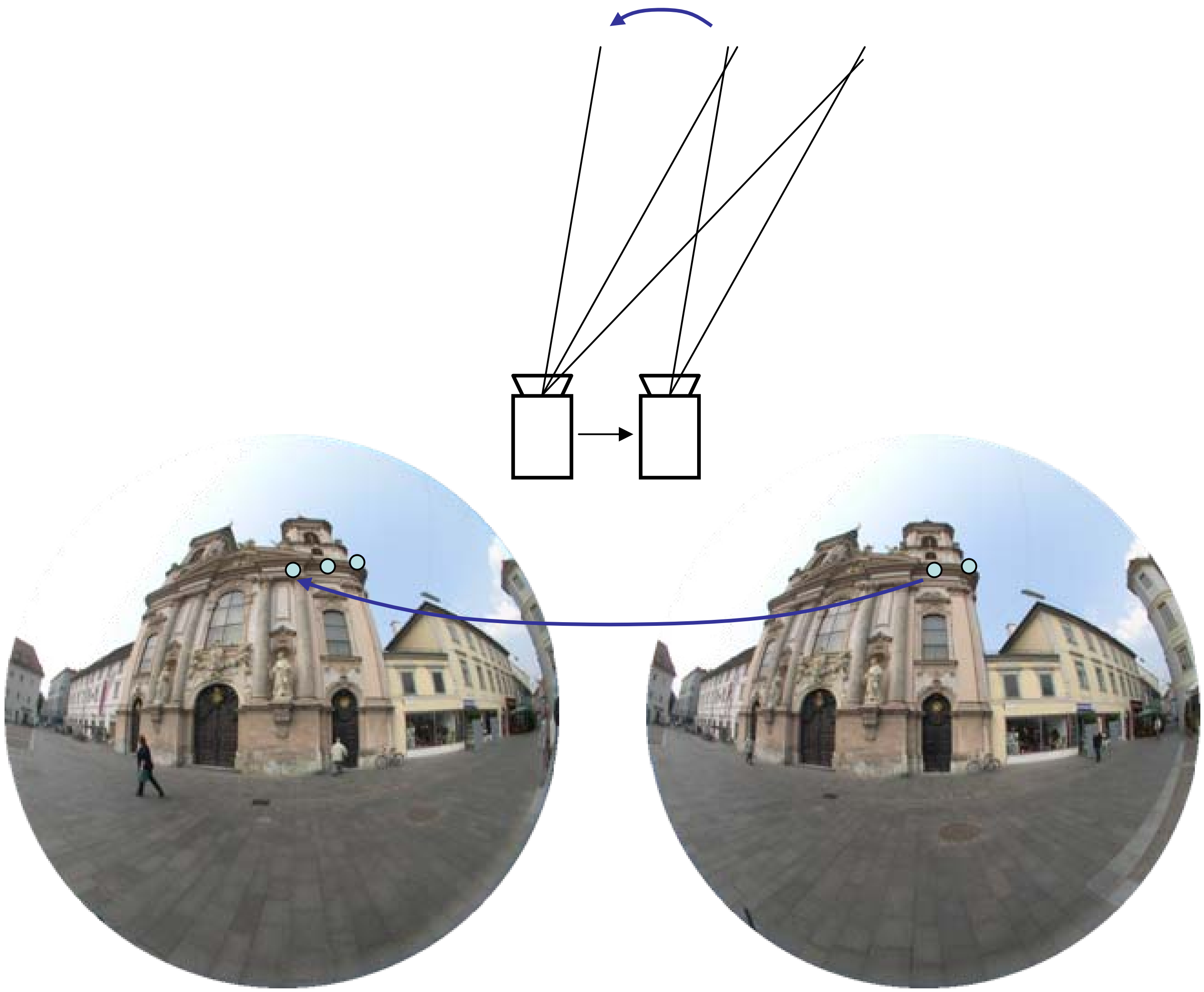
Non-parametric self-calibration

Flow curves for pure translations



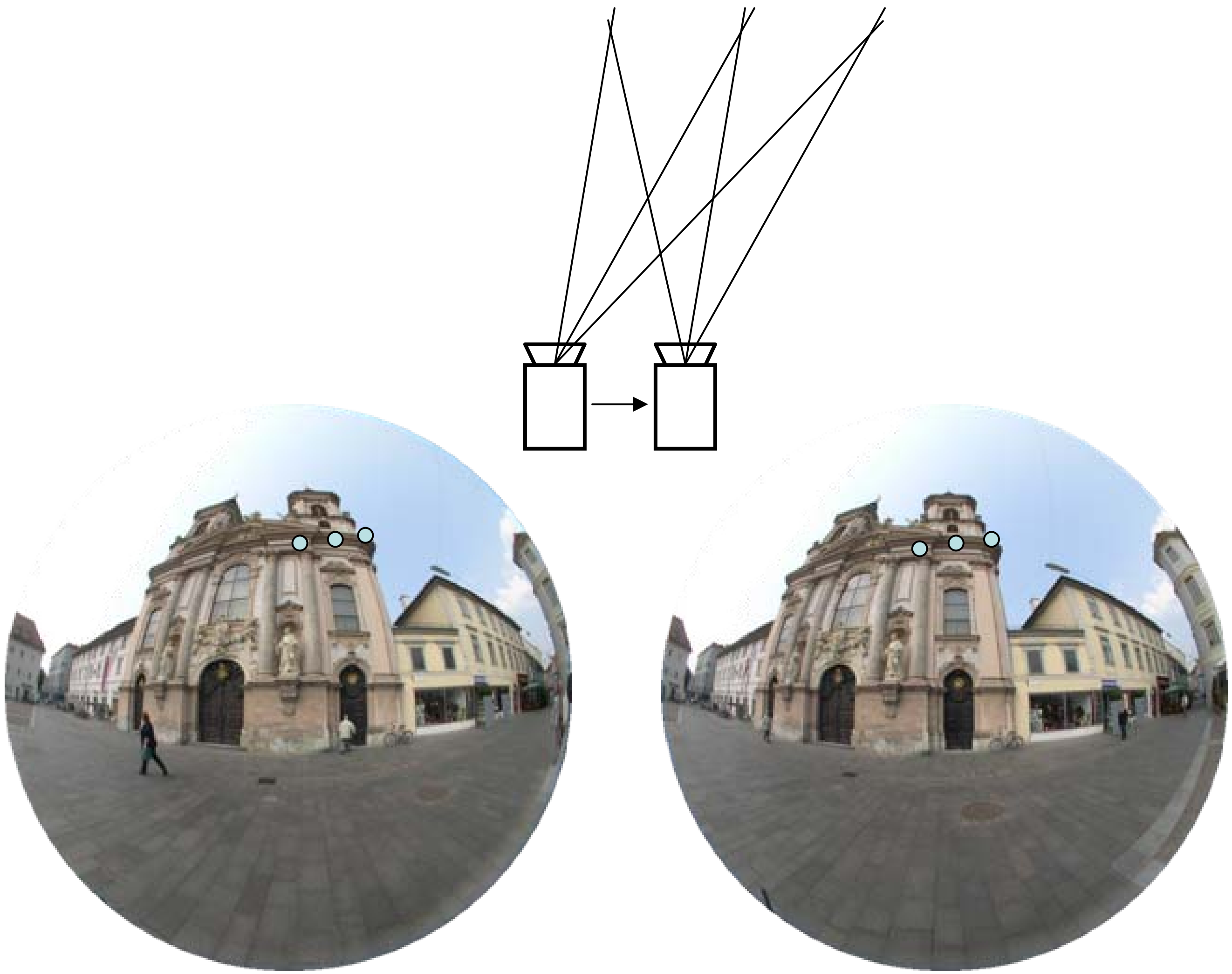
Non-parametric self-calibration

Flow curves for pure translations



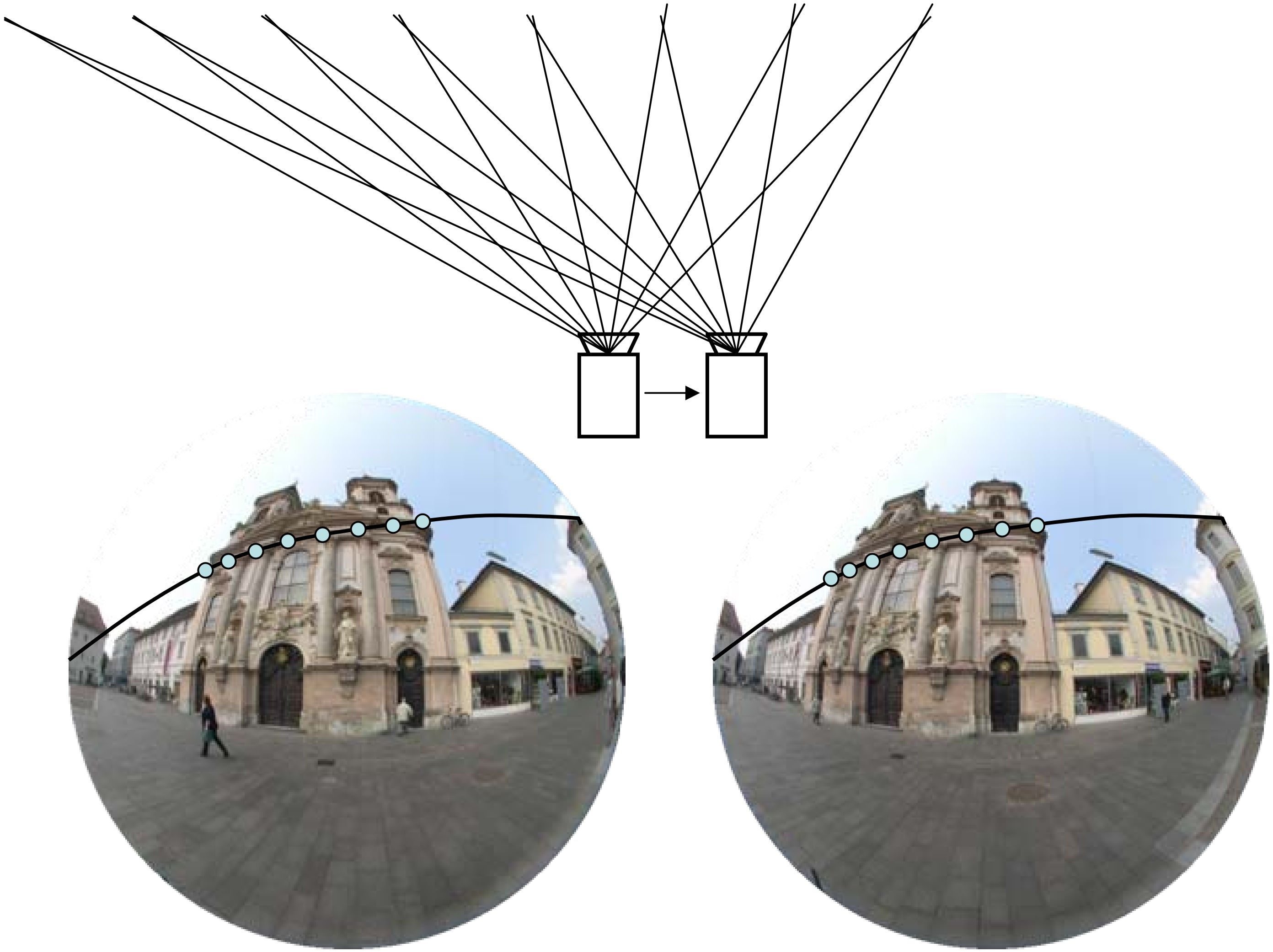
Non-parametric self-calibration

Flow curves for pure translations



Non-parametric self-calibration

Flow curves for pure translations



Non-parametric self-calibration

Flow curves for pure translations

- They actually are epipolar curves…
- Can be obtained from one image pair, but also from image sequence of course
- Provide the following information on calibration:
  - projection rays associated with pixels on a flow curve, are coplanar
- Flow curves for several translational motions give several coplanarity constraints, that allow to do self-calibration

Non-parametric self-calibration

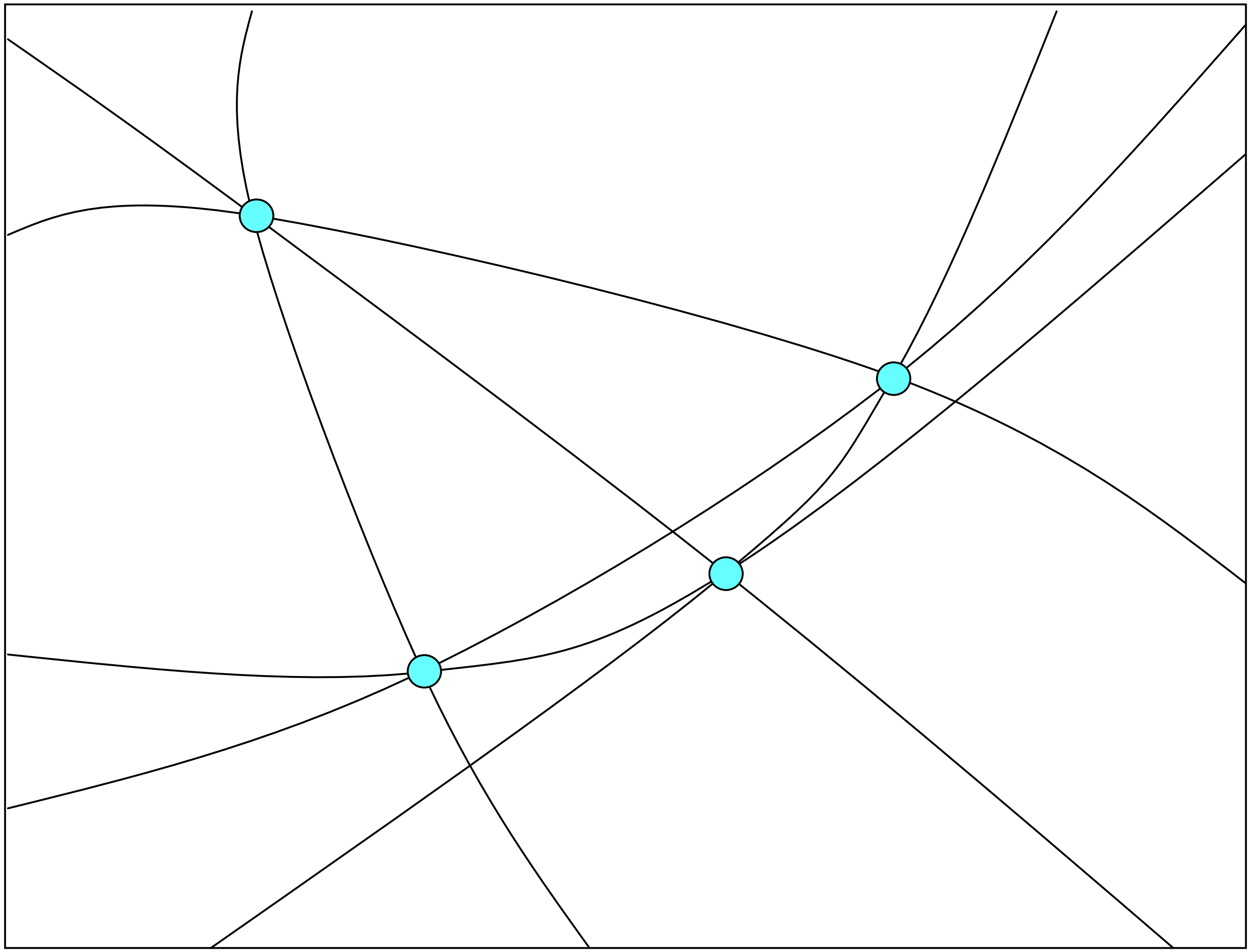
Self-calibration from several translational motions:

- Goal: compute directions of projection rays (their points at infinity)
- Coplanarity of projection rays ≡ collinearity of points at infinity
- We have many collinearity constraints (one per flow curve)
- Collinearity is invariant to projective transformations

  → ray directions can be computed only up to a projective transformation

Non-parametric self-calibration

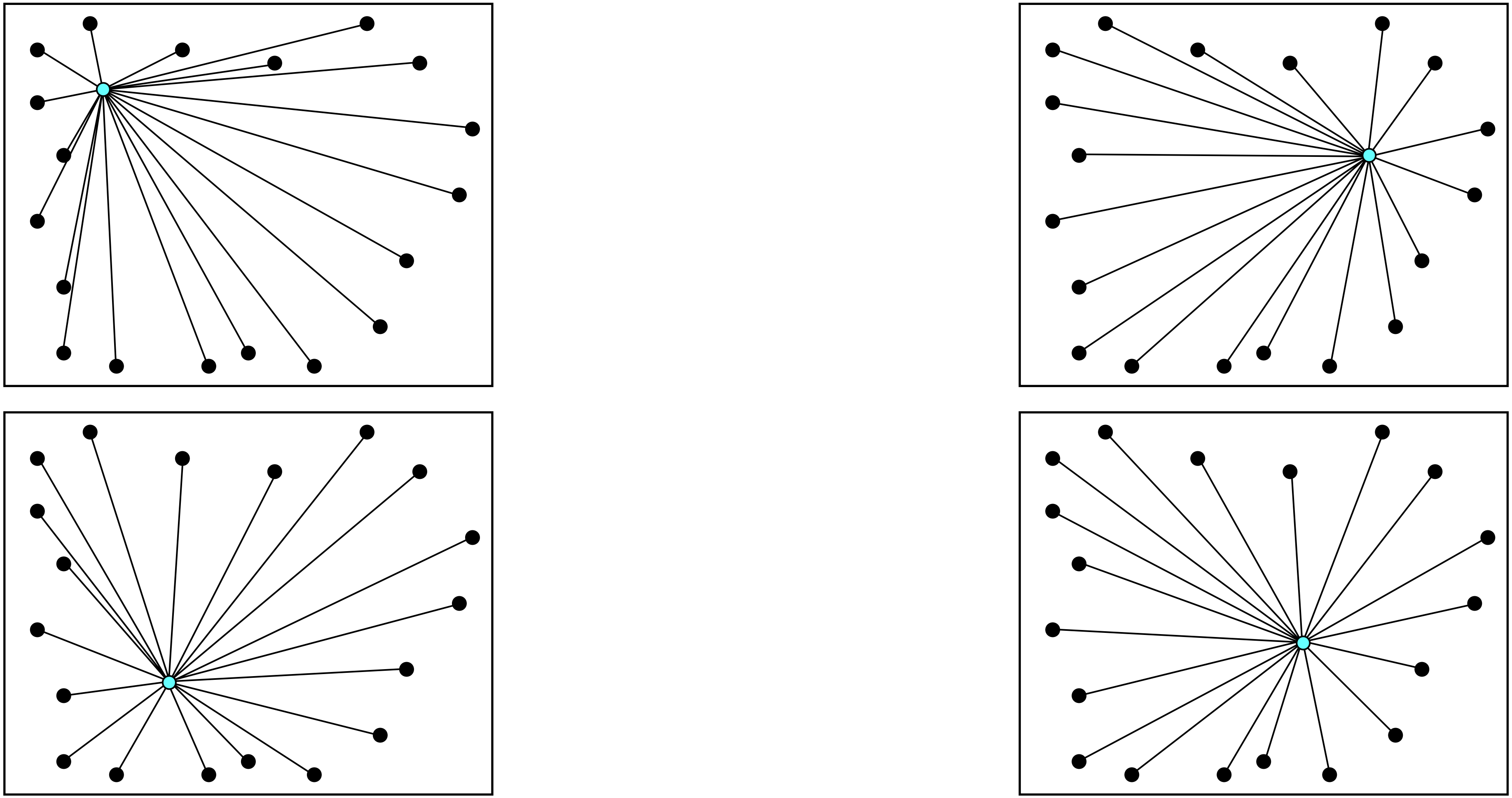
Non-perspective cameras: flow curves not straight, but the following algorithm can be applied without changes (but is difficult to illustrate…)



Non-parametric self-calibration

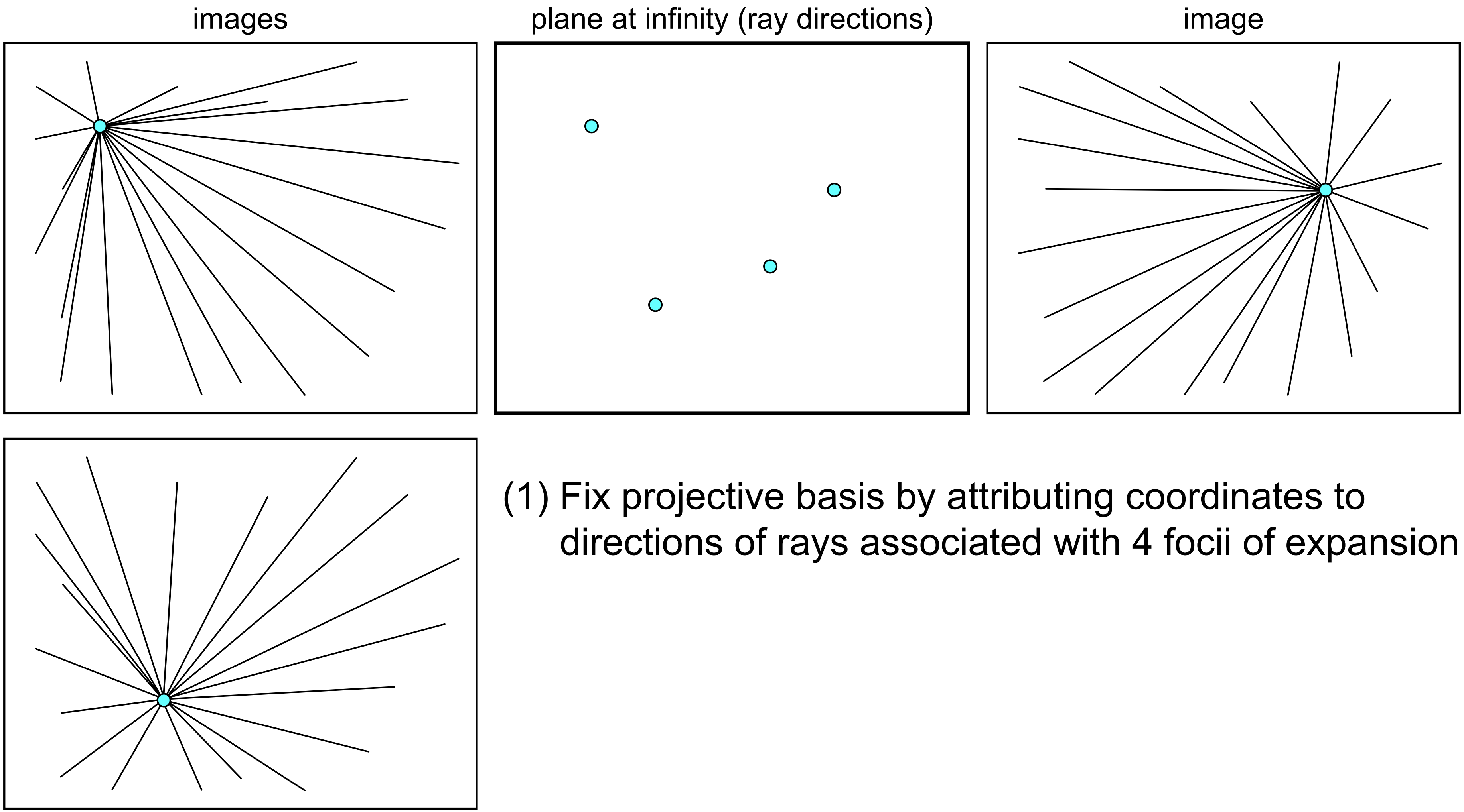
Illustration for perspective camera:

Flow curves for 4 translational motions, with focii of expansion



Non-parametric self-calibration

Illustration of algorithm idea for perspective camera:

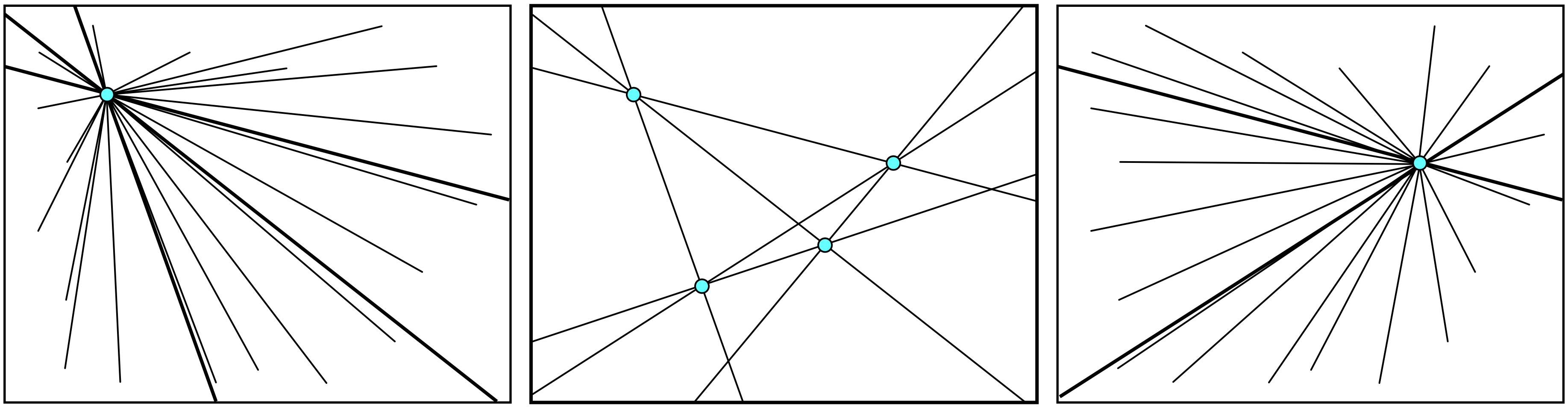


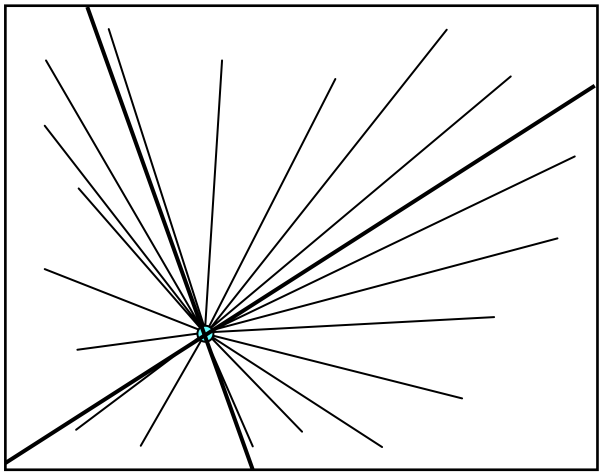
(1) Fix projective basis by attributing coordinates to directions of rays associated with 4 focii of expansion

Non-parametric self-calibration

Illustration of algorithm idea for perspective camera:

images — plane at infinity (ray directions) — image





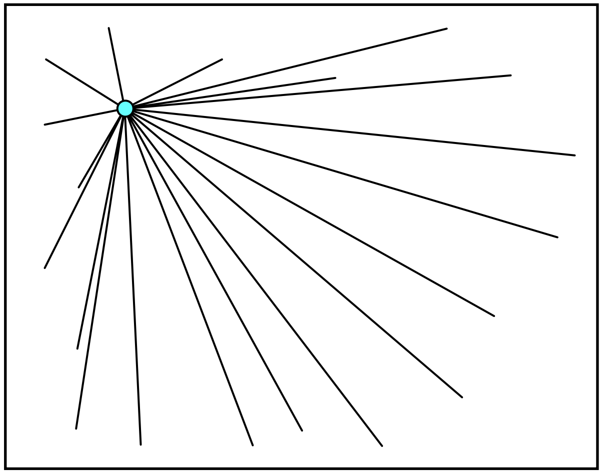
(1) Fix projective basis by attributing coordinates to directions of rays associated with 4 focii of expansion

(2) Compute lines at infinity for flow curves with two known ray directions

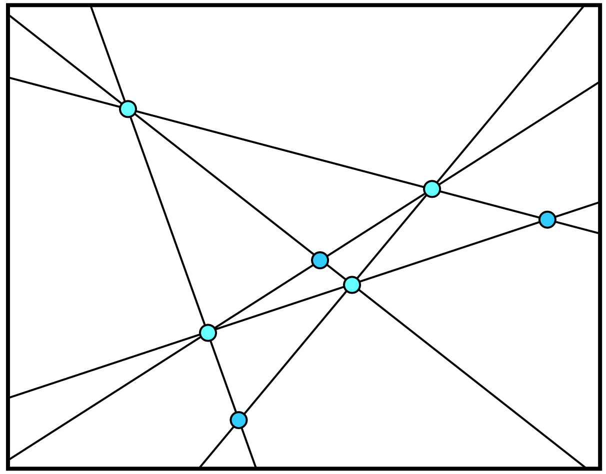
Non-parametric self-calibration

Illustration of algorithm idea for perspective camera:

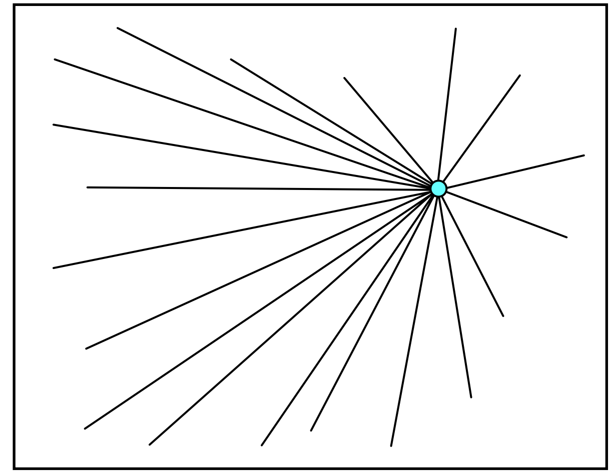
images

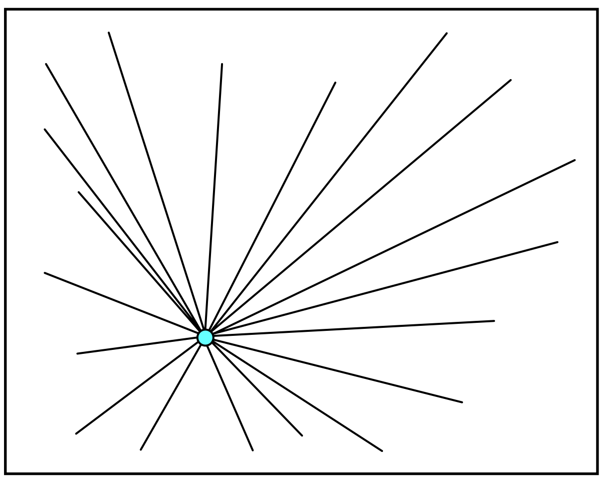


plane at infinity (ray directions)



image





(1) Fix projective basis by attributing coordinates to directions of rays associated with 4 focii of expansion

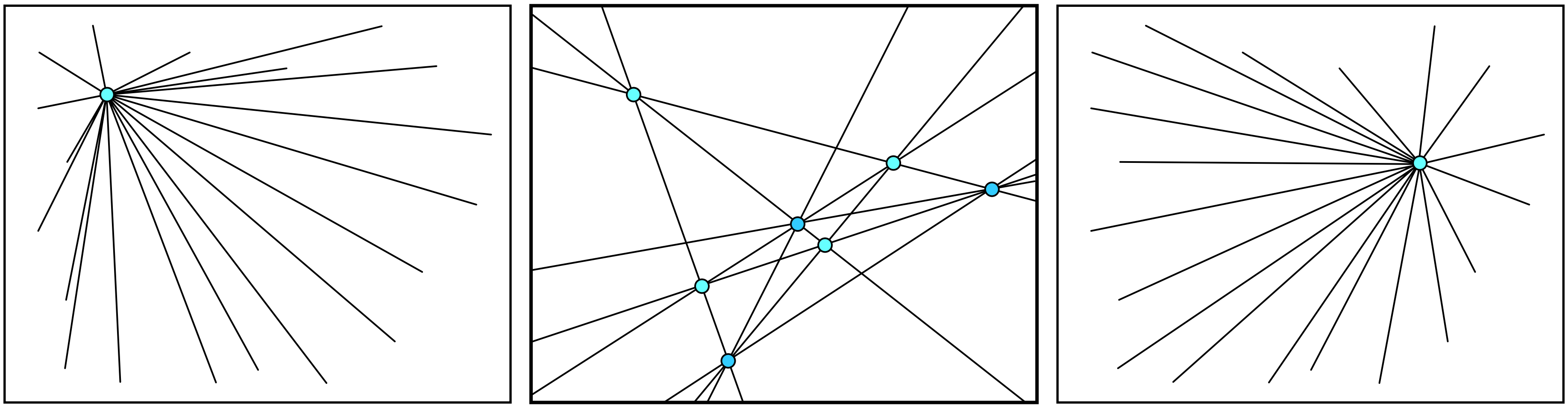
(2) Compute lines at infinity for flow curves with two known ray directions

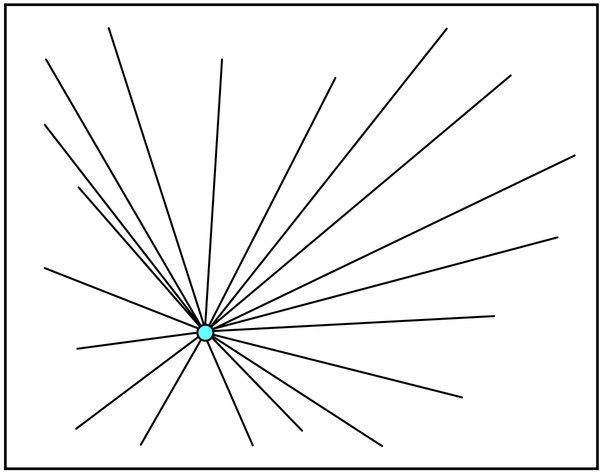
(3) Compute directions of rays lying on two flow curves with known line at infinity

(4) Go to (2) until “convergence”

Non-parametric self-calibration

Illustration of algorithm idea for perspective camera:





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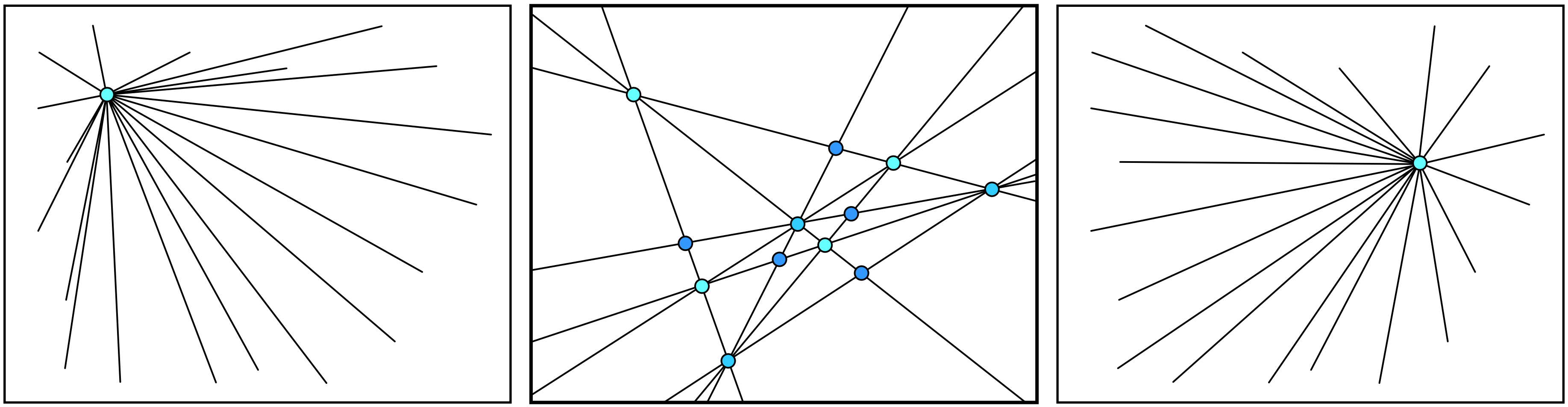
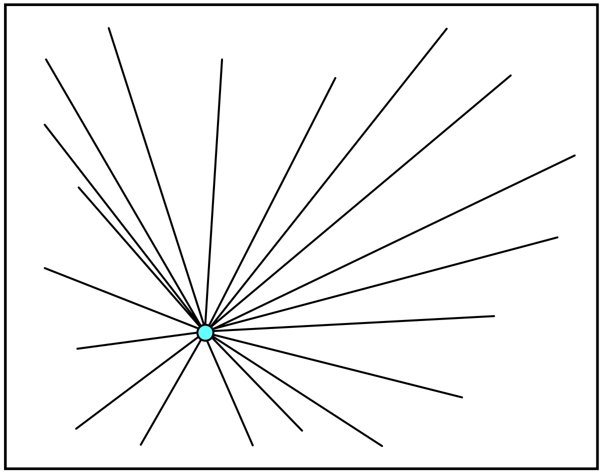
Non-parametric self-calibration

Illustration of algorithm idea for perspective camera:

images | plane at infinity (ray directions) | image





(1) Fix projective basis by attributing coordinates to directions of rays associated with 4 focii of expansion

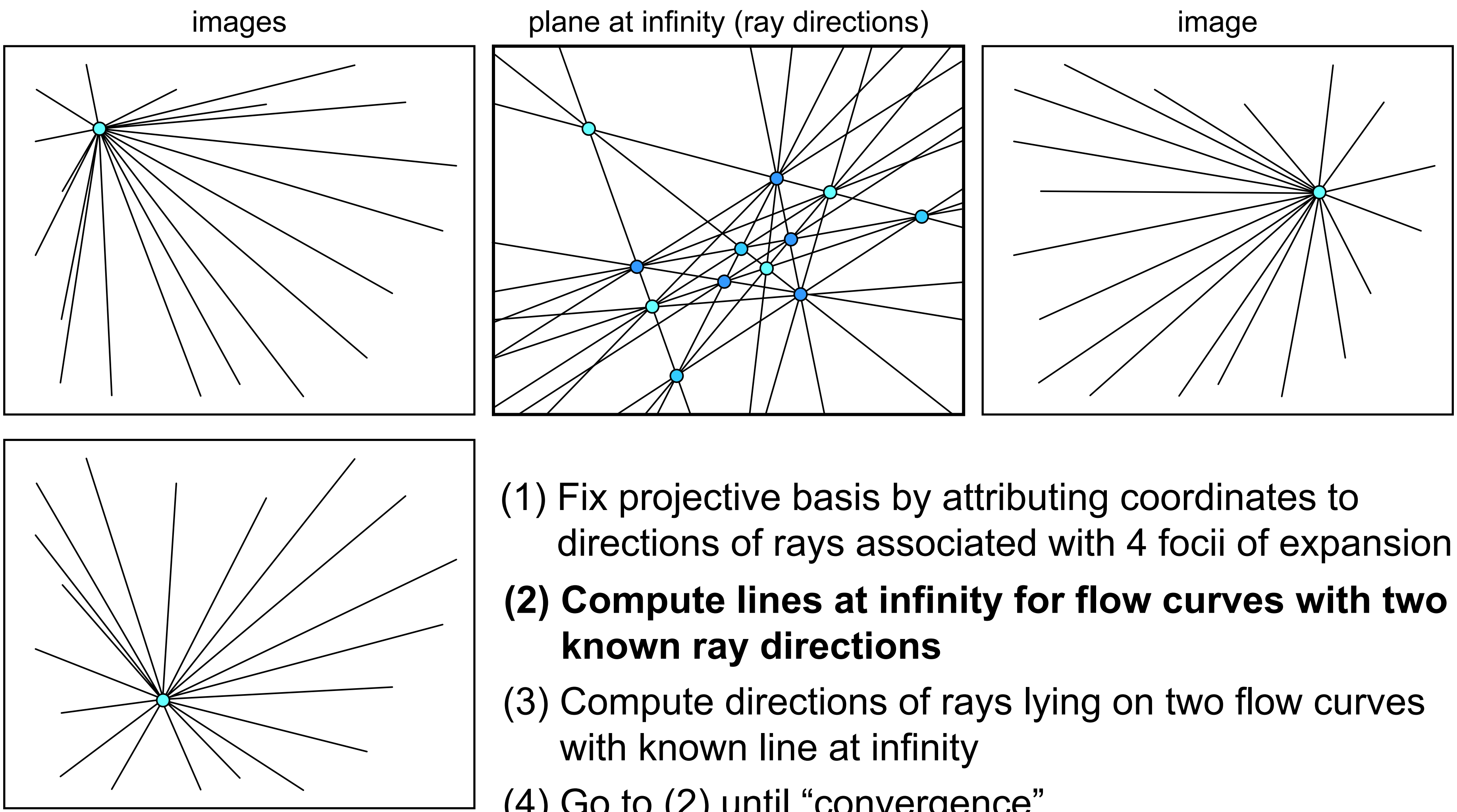
(2) Compute lines at infinity for flow curves with two known ray directions

(3) Compute directions of rays lying on two flow curves with known line at infinity

(4) Go to (2) until “convergence”

Non-parametric self-calibration

Illustration of algorithm idea for perspective camera:



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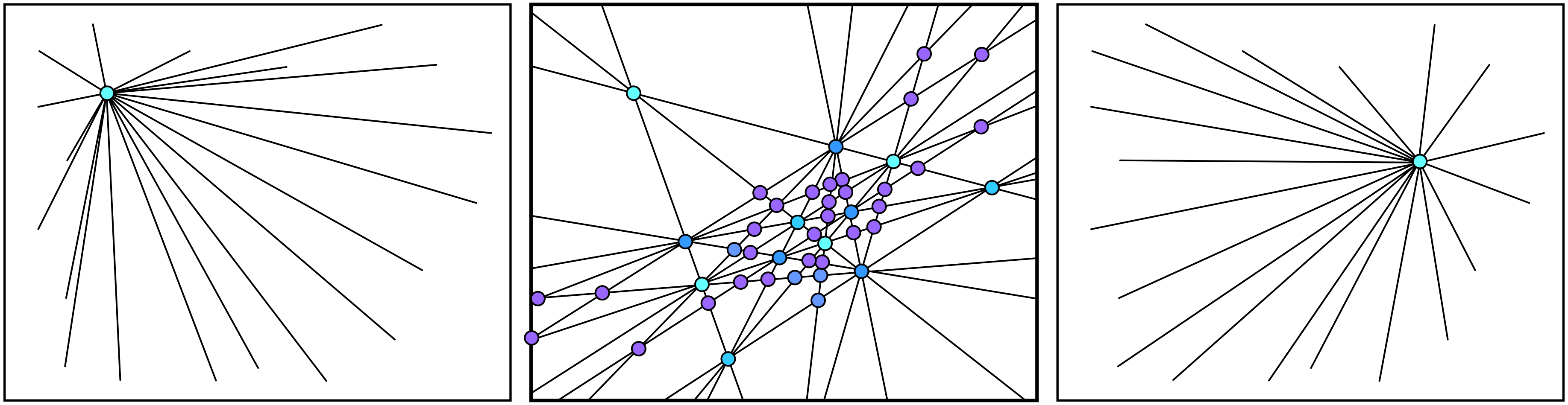

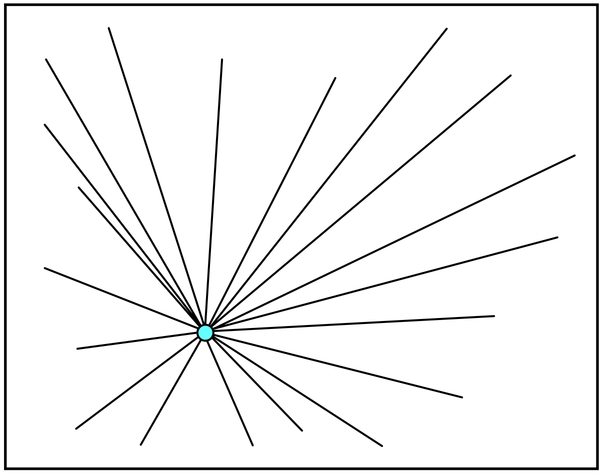

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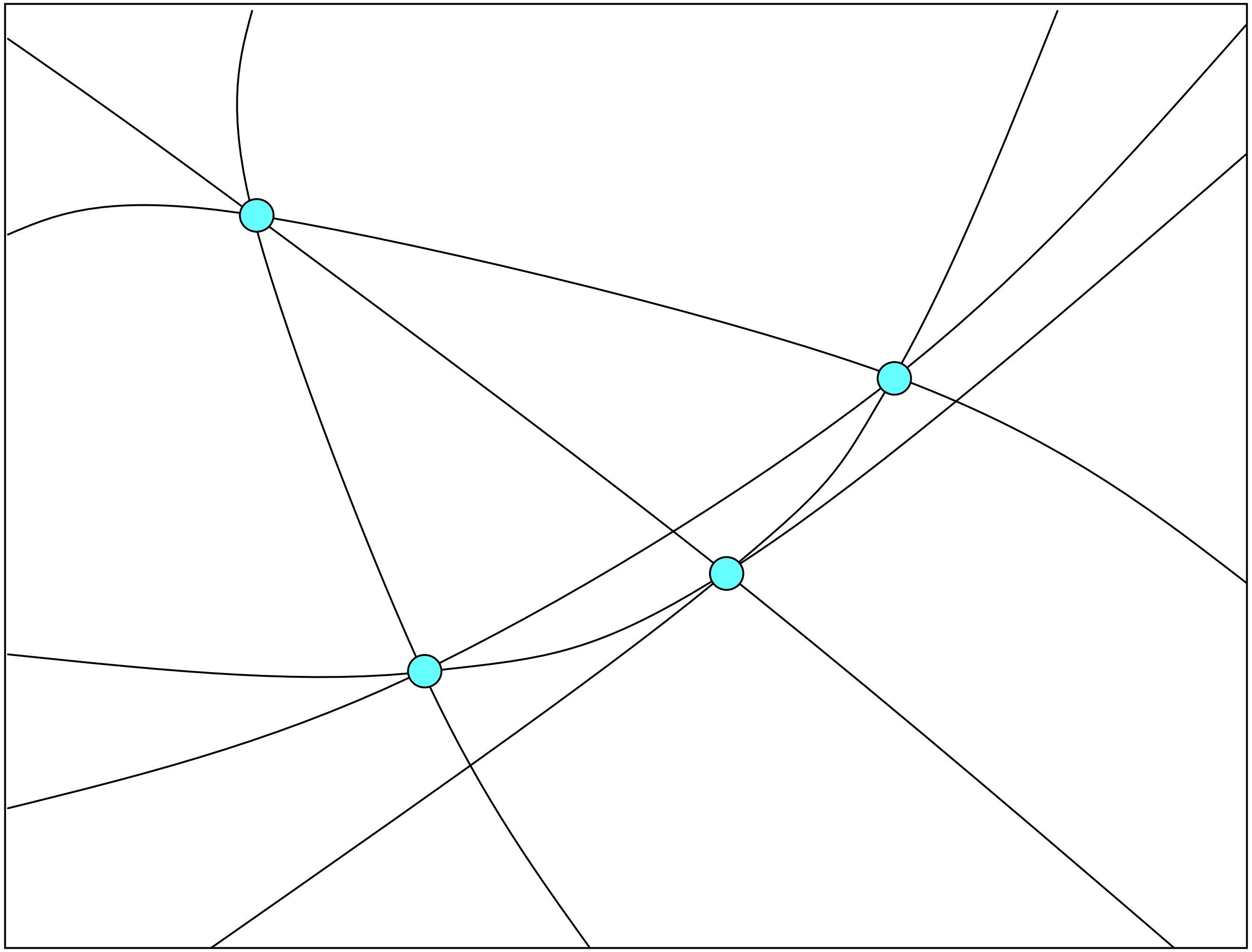
Non-parametric self-calibration

Reminder: non-perspective cameras: flow curves not straight, but same algorithm can be applied (but is difficult to illustrate…)



Non-parametric self-calibration

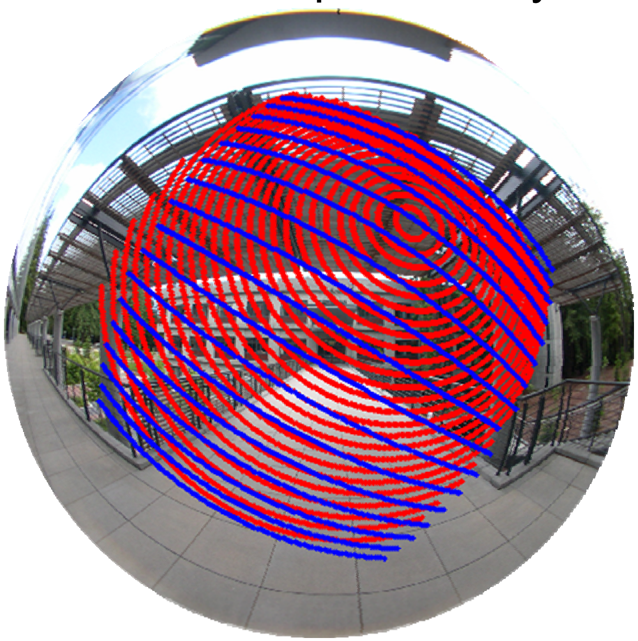
Self-calibration from several translational motions:

- Ray directions can be computed up to a projective transformation

  → amount of calibration knowledge is now equivalent to that of an uncalibrated perspective camera

  → any self-calibration method for perspective cameras can be applied to complete the self-calibration

Complete self-calibration is possible by doing translational and rotational motions

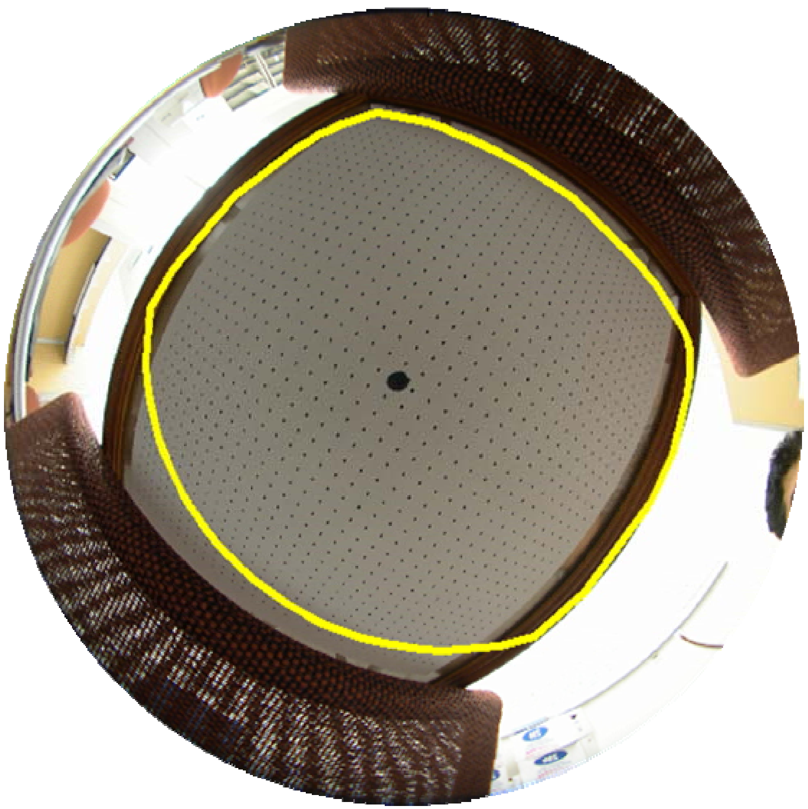


Non-parametric self-calibration

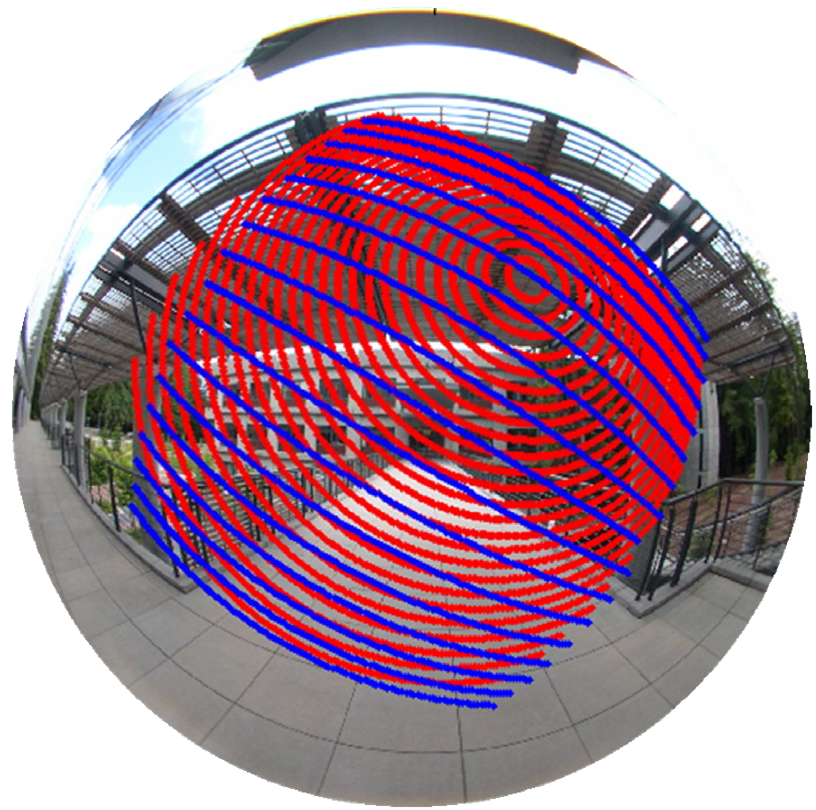
Complete self-calibration is possible by doing translational and rotational motions

- In first experiments, we used images of a calibration grid
  (just for tracking and computing flow curves)

One input image and
calibrated region

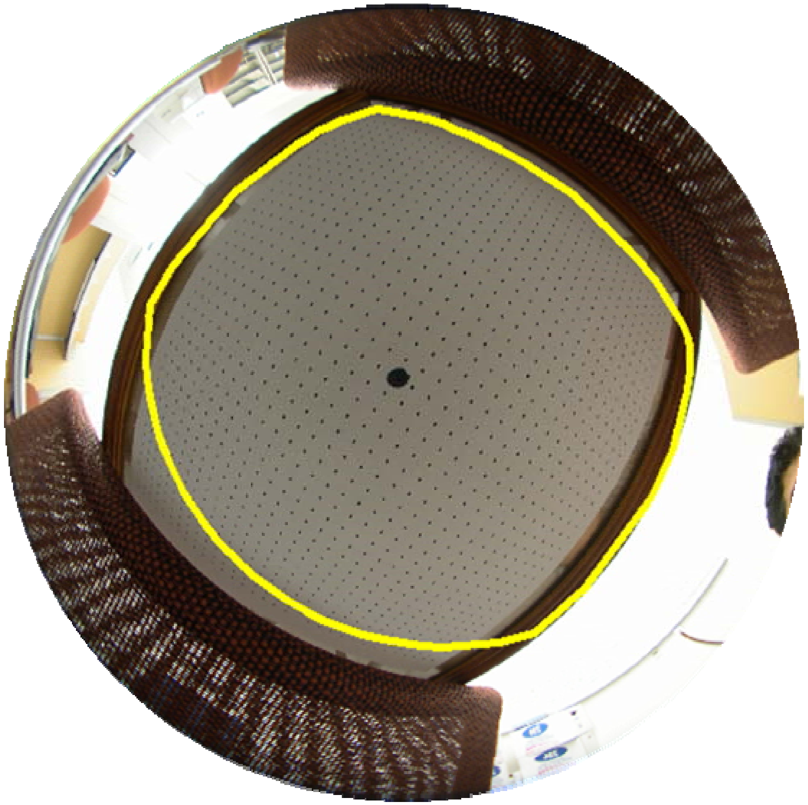


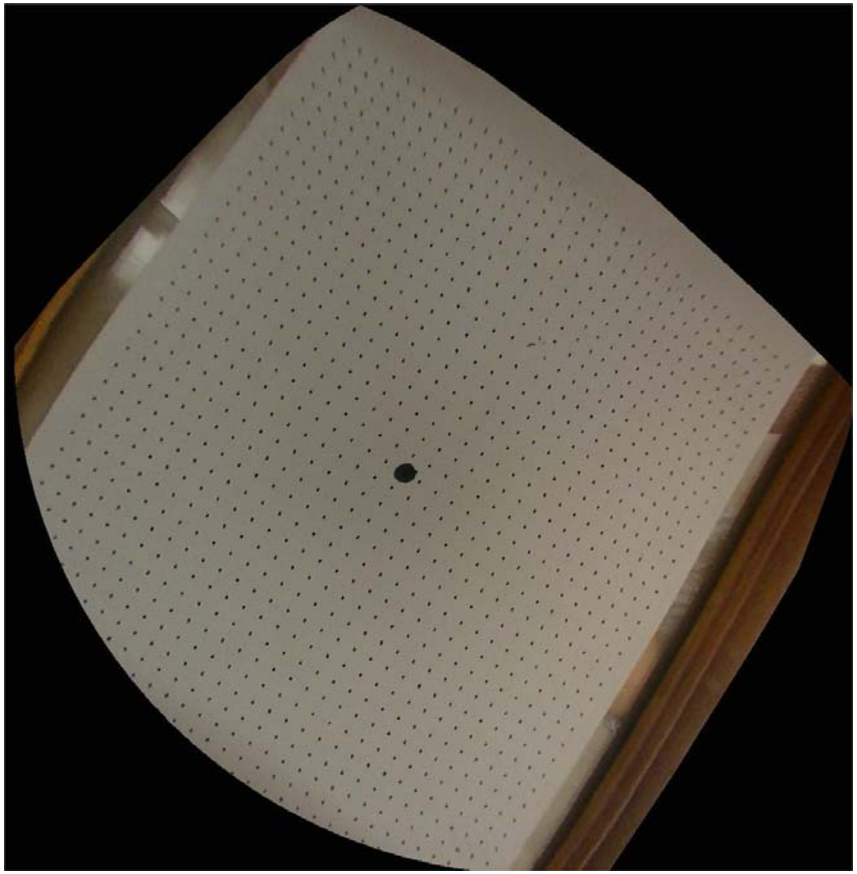
Display of flow curves
on some other image



Non-parametric self-calibration

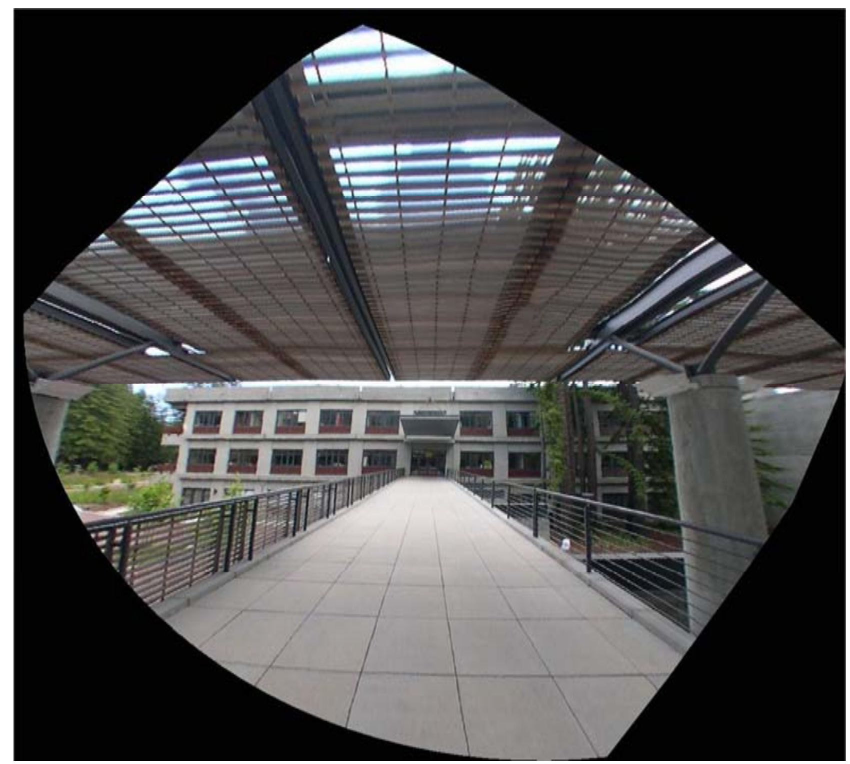
Result of distortion correction using self-calibration result:





Non-parametric self-calibration

Result of distortion correction using self-calibration result:



Non-parametric self-calibration

Result of distortion correction using self-calibration result:

Summary on non-parametric calibration:

- Approaches allowing to calibrate any camera:
  compute projection ray for each pixel (or for other discretization)

- Tradeoff:
  - generality of camera model (need fewer algorithms, potential accuracy)
  - stability (may need many images for calibrating of non-central cameras)

- Good results for radially symmetric and central cameras;
  also for some non-central cameras (multi-camera systems, misaligned catadioptric cameras)

- Self-calibration is possible but remains difficult

- Theoretical study of self-calibration requires continuous camera model

Summary on non-parametric calibration:

- Generic imaging model gives **backprojection**:
  - for pixels, backprojection is given by the lookup table
  - for other points, backprojection can be easily obtained using some interpolation of rays associated with neighboring pixels

- **Projection** is more problematic, but can be done, e.g.:
  - Finding closest rays to a 3D point and determining image point by interpolating positions of pixels associated to these rays

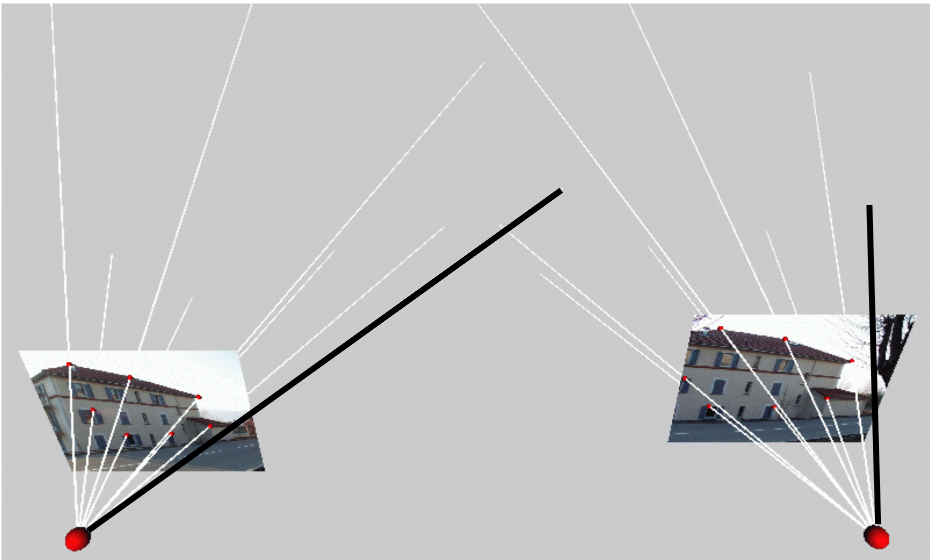
Contents

- Introduction
- General imaging models
- Non-parametric calibration and distortion correction
- Non-parametric self-calibration
- Structure-from-motion

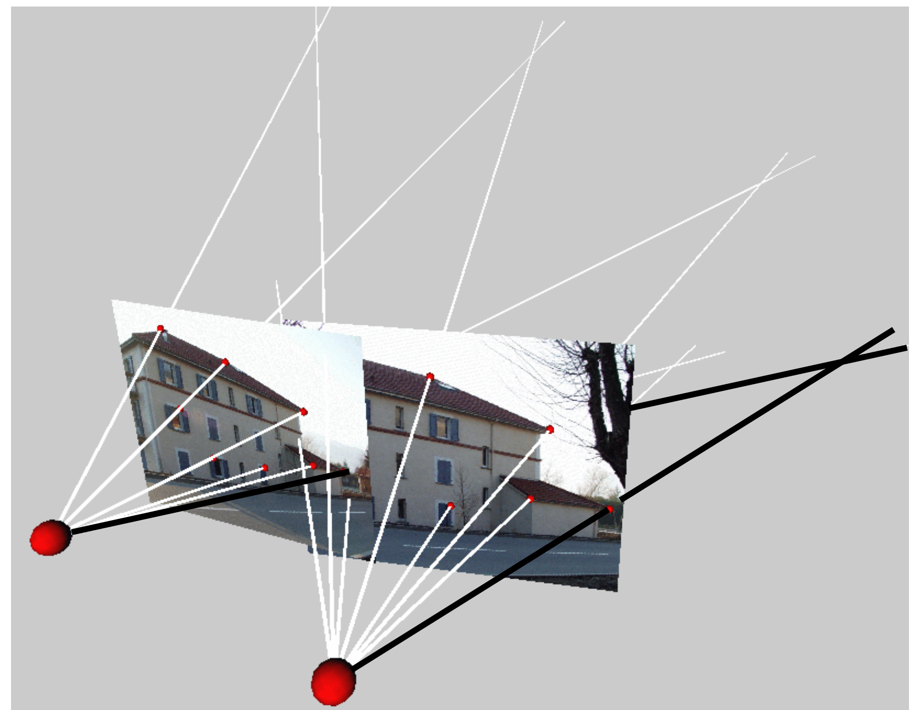
Motivation:

- Many different SfM algorithms (pose, motion, triangulation, …) exist, for different camera types
- But, in principle, if calibrated cameras are considered, one single approach for each SfM problem is sufficient, for all camera types

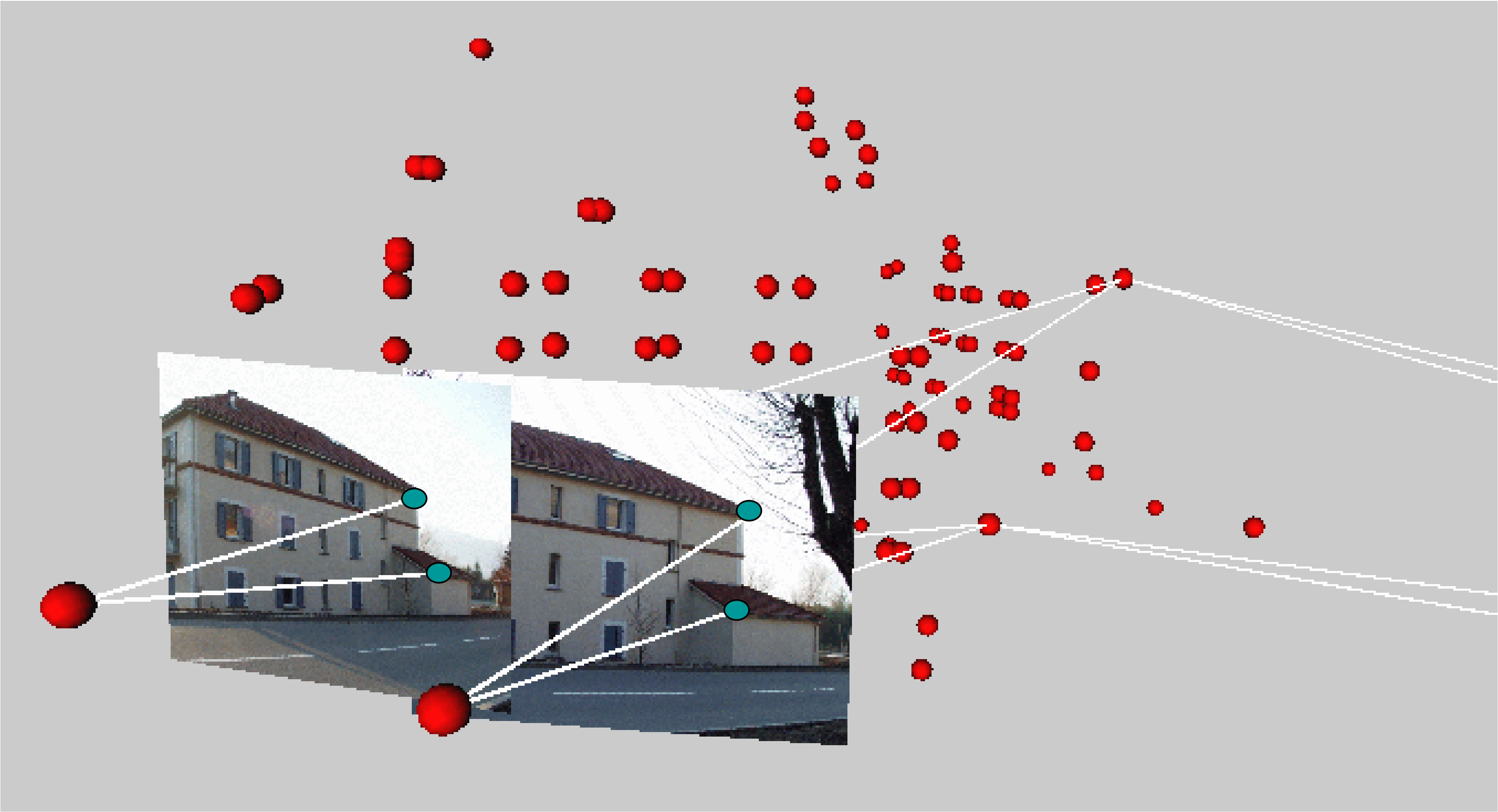
Calibration: determine, for each pixel, the corresponding line of sight ("projection ray")



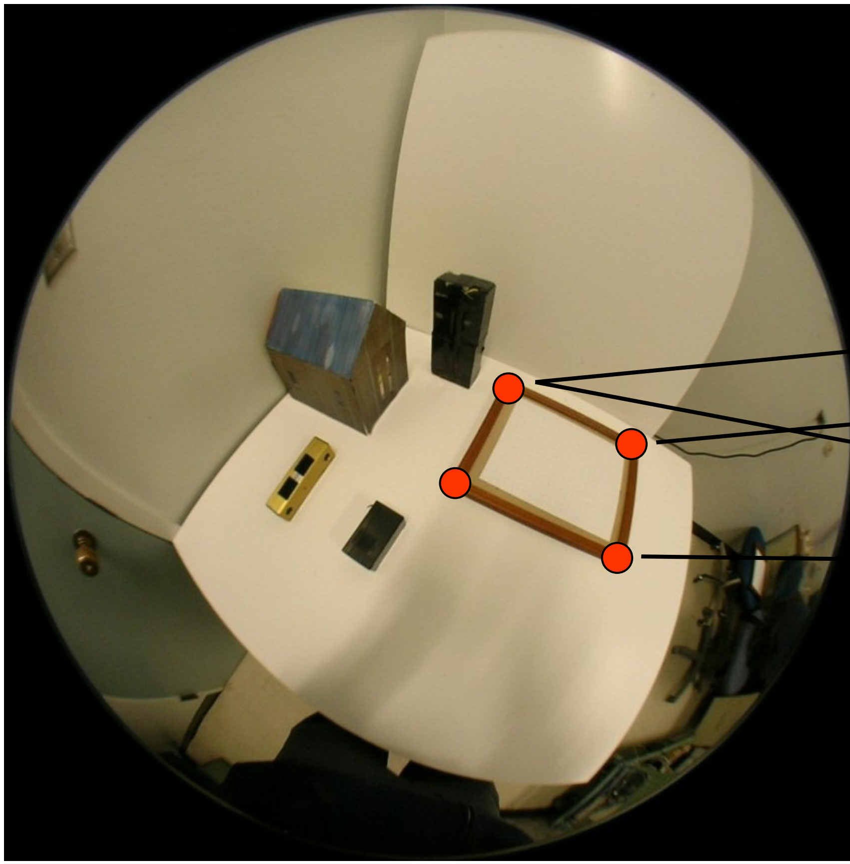
Motion estimation: compute motion such that matching rays intersect

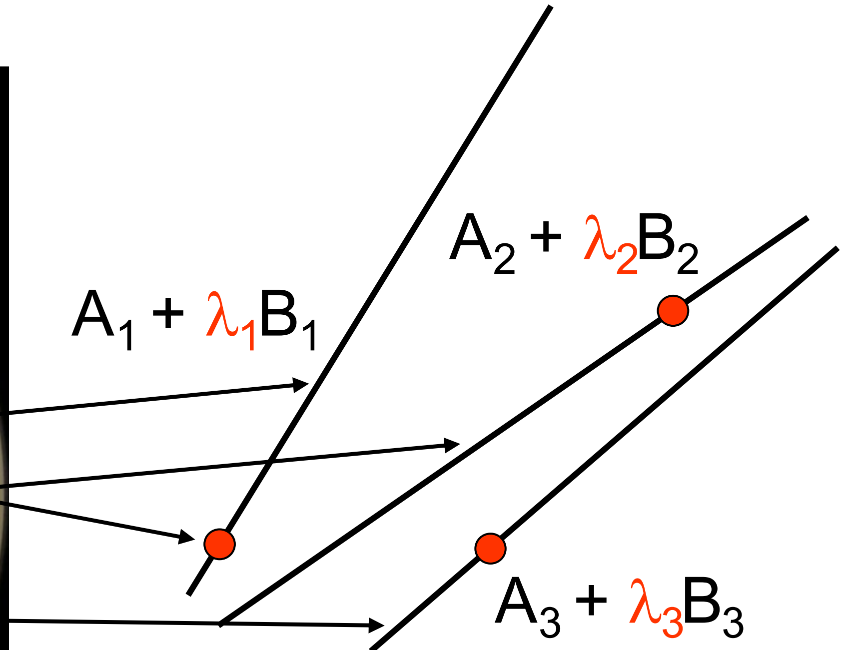


Triangulation / 3D Reconstruction



Pose estimation of known object



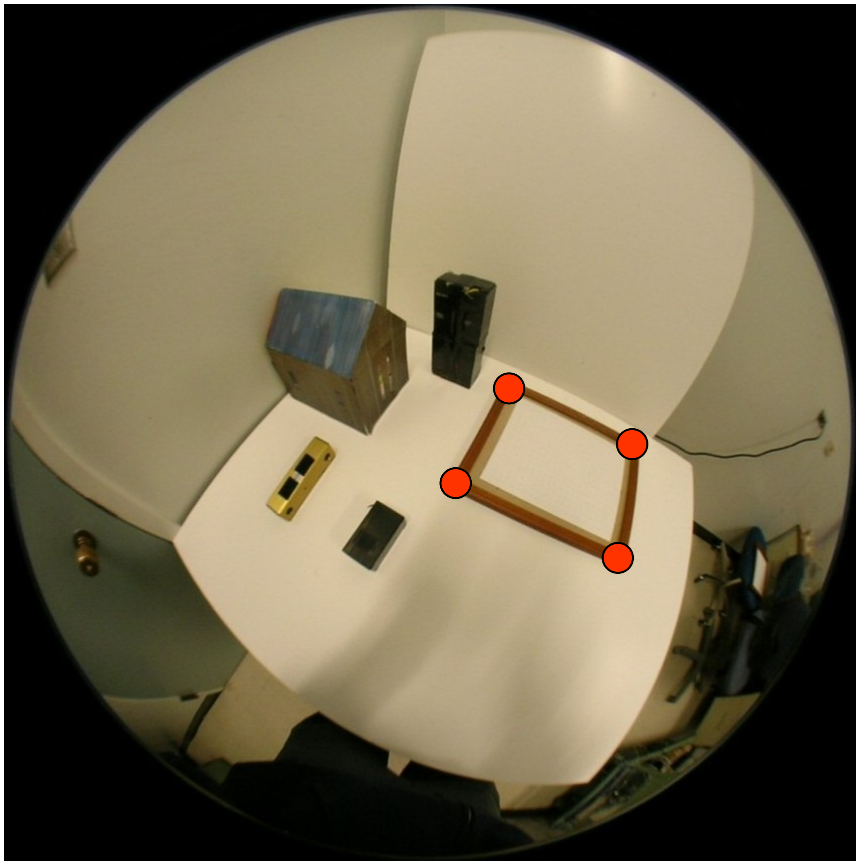


$$\mathrm{dist}^2(A_1 + \lambda_1 B_1 \,,\, A_2 + \lambda_2 B_2) = d_{12}^2$$

$$\mathrm{dist}^2(A_1 + \lambda_1 B_1 \,,\, A_3 + \lambda_3 B_3) = d_{13}^2$$

$$\mathrm{dist}^2(A_2 + \lambda_2 B_2 \,,\, A_3 + \lambda_3 B_3) = d_{23}^2$$

Pose estimation of known object



- 3 quadratic equations: up to 8 solutions
- Central camera: solutions come in mirrored pairs (for a solution in front of the camera, another one behind exists too)
- Non-central camera: no such simple symmetry exists
- With 4 points, unique solution in general

[Chen-Chang-PAMI’04,Nistér-CVPR’04,Ramalingam-etal-OMNIVIS’04]

Motion estimation: unknown scene

- Pixel matches gives rise to ray matches
- Represent rays using Plücker coordinates
- Displacement for Plücker coordinates:

$$L'_1 = \begin{pmatrix} R & 0 \\ -[t]_\times R & R \end{pmatrix}_{6\times6} L_1$$

- Rays intersect if

$$L_2^T \begin{pmatrix} 0 & Id \\ Id & 0 \end{pmatrix}_{6\times6} L'_1 = 0$$

$\rightarrow$

Essential matrix

$$E = \begin{pmatrix} -[t]_\times R & R \\ R & 0 \end{pmatrix}_{6\times6}$$

$$L_2^T \, E \, L_1 = 0$$

Motion estimation:

(1) Estimation of E (possible using linear equations: minimum 17 matches)

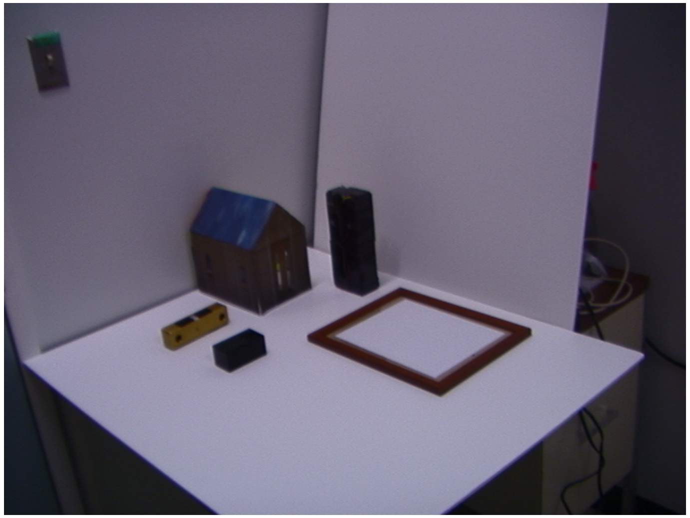
(2) Extraction of R and t from E (simple)

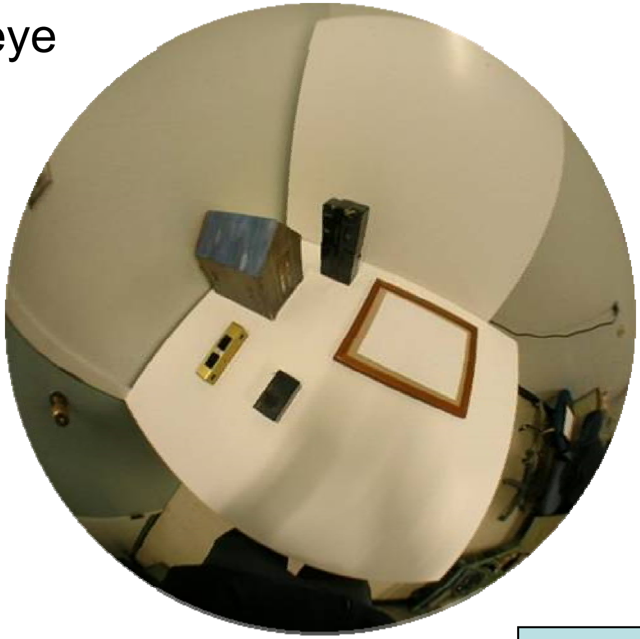
Note: scale of motion can be estimated if non-central cameras!
(but may be unreliable if cameras not very non-central)

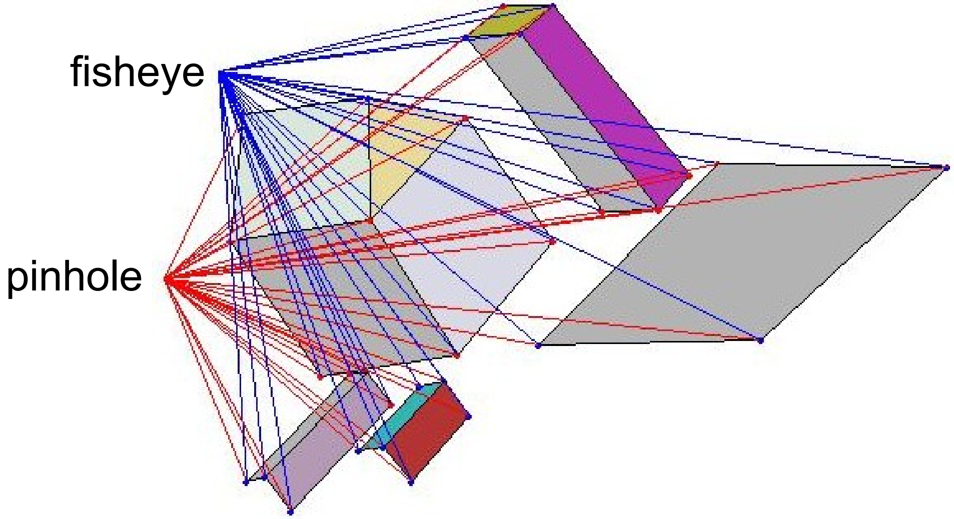
Variants for: axial, x-slit, central cameras

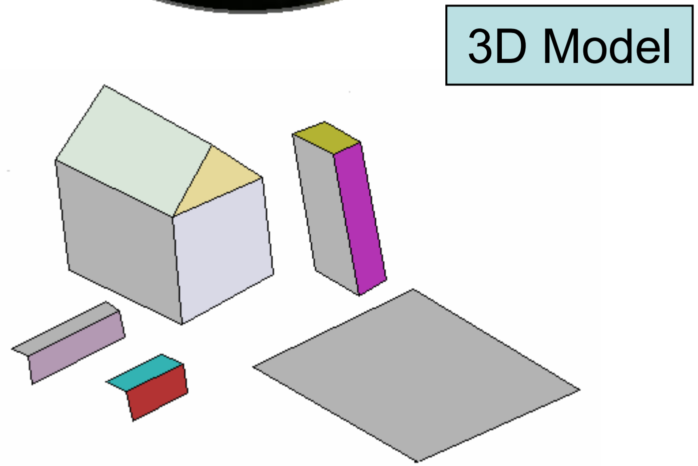
[Pless-CVPR’03,Sturm-etal-Bookchapter’06]

Motion estimation and 3D from pinhole+fisheye

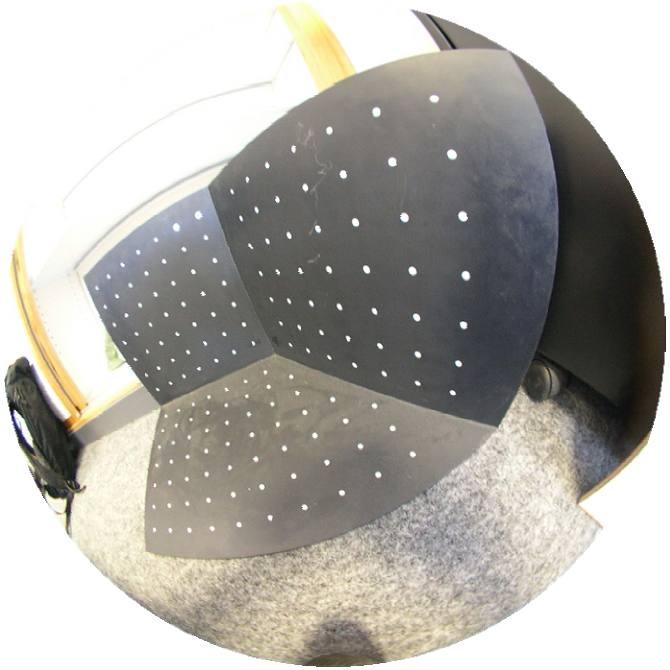








Motion estimation and 3D from pinhole+fisheye



3D Model

Perspective epipolar geometry:

- Epipolar line of a pixel p computed via the fundamental matrix: $v=Fp$

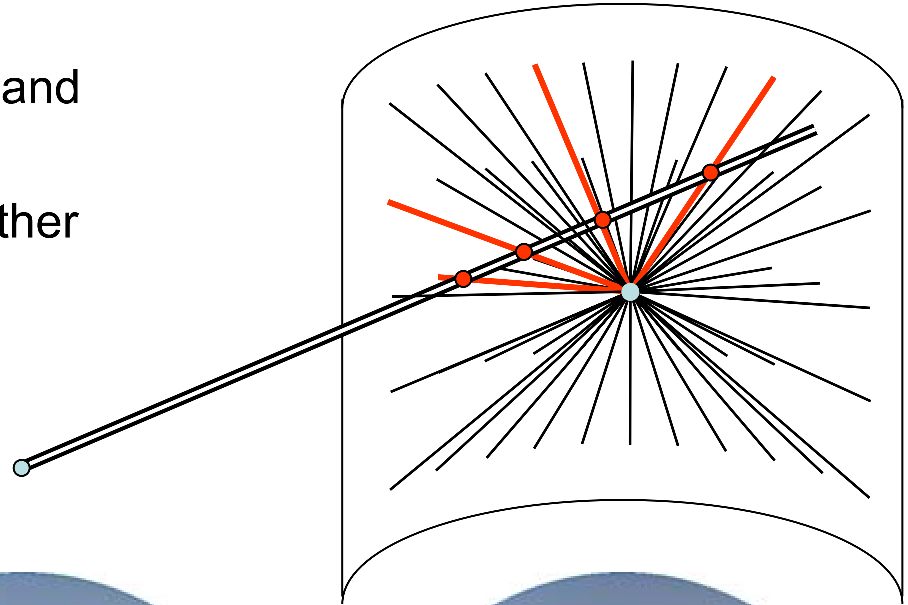
Such a parametric epipolar geometry exists for some omnidirectional cameras, e.g. para-catadioptric ones

It also exists between cameras of different types, e.g. a stereo pair consisting of a perspective and a para-catadioptric camera

[Svoboda-etal-ECCV'98,Feldman-et-al-ICCV'05,Sturm-OMNIVIS'02]

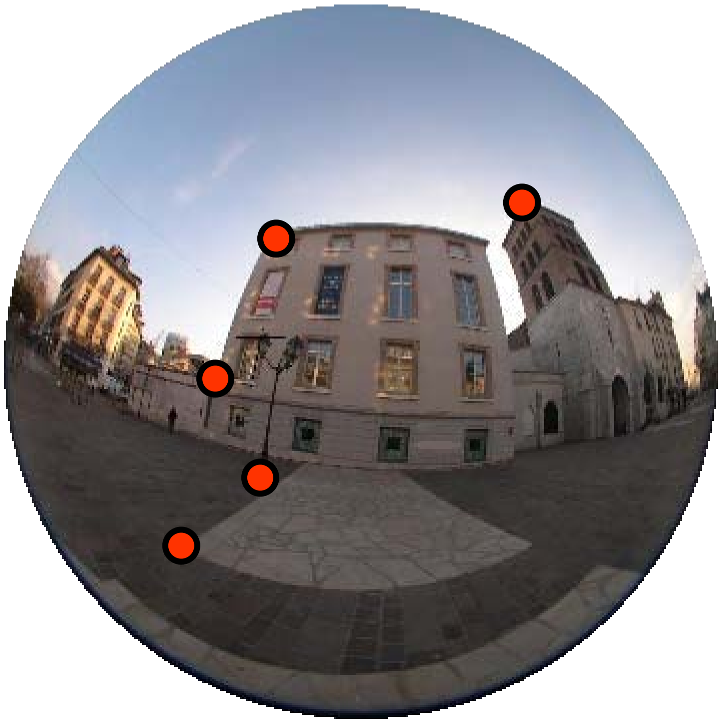
Non-parametric epipolar geometry:

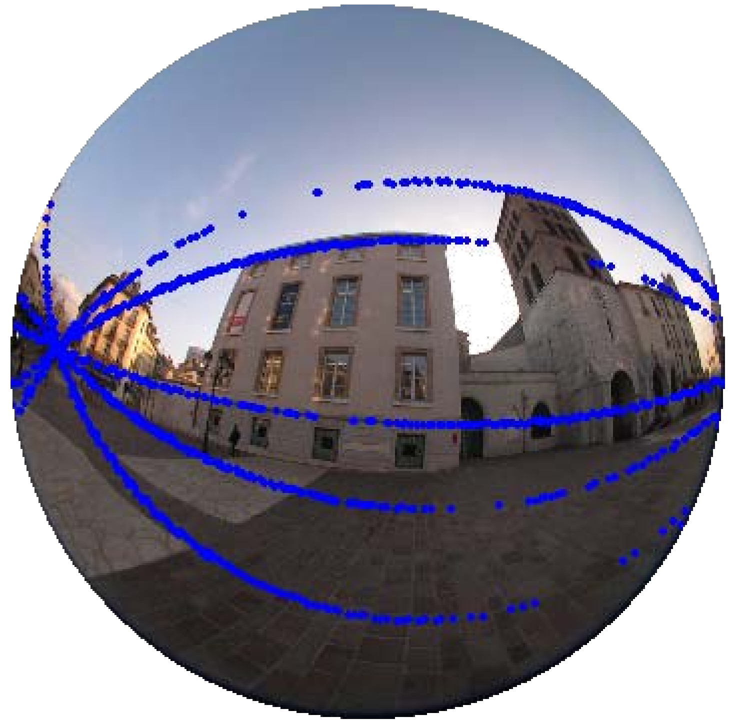
- Consider a pixel in one image and the associated projection ray
- Determine projection rays of other camera that cut that ray
- The associated pixels form an “epipolar curve”



Here: illustration with central cameras, but concept is applicable to whatever camera, i.e. also non-central ones

Non-parametric epipolar geometry:





Multi-view geometry for perspective images:

- Consider points (or other features) in images
- Which geometric constraints exist that tell if points are potential matches?
  - 2 images: epipolar geometry (fundamental/essential matrix)

$$q_2^T \mathbf{E}\, q_1 = 0$$

  - 3 or 4 images: trifocal and quadrifocal tensors

$$\sum_{i_1=1}^{3} \sum_{i_2=1}^{3} \cdots \sum_{i_n=1}^{3} q_{1,i_1} q_{2,i_2} \cdots q_{n,i_n} T_{i_1,i_2,\cdots,i_n} = 0$$

Multi-view geometry for generic imaging model:

- Constraints between *projection rays*

$$\sum_{i_1=1}^{6} \sum_{i_2=1}^{6} \cdots \sum_{i_n=1}^{6} L_{1,i_1} L_{2,i_2} \cdots L_{n,i_n} T_{i_1,i_2,\cdots,i_n} = 0$$

Perspective multi-view geometry:

- Consider points $\mathbf{q}_i$ in *n* images with projection matrices P_i
- They are potential matches if scalars λ_i and a 3D point $\mathbf{Q}$ exist with:

$$\lambda_i \mathbf{q}_i = \mathsf{P}_i \mathbf{Q}, \quad \forall i = 1 \cdots n$$

- This can be written as:

$$\underbrace{\begin{pmatrix} \mathsf{P}_1 & \mathbf{q}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathsf{P}_2 & \mathbf{0} & \mathbf{q}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathsf{P}_n & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{q}_n \end{pmatrix}}_{\mathsf{M}} \begin{pmatrix} \mathbf{Q} \\ -\lambda_1 \\ -\lambda_2 \\ \vdots \\ -\lambda_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

- Existence of null-vector implies rank-deficiency of M
- M is of size 3n × 4+n
  → all submatrices (4+n) × (4+n) have zero determinant

$$\underbrace{\begin{pmatrix} \mathsf{P}_1 & \mathbf{q}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathsf{P}_2 & \mathbf{0} & \mathbf{q}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathsf{P}_n & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{q}_n \end{pmatrix}}_{\mathsf{M}}$$

- Determinants of submatrices can be written as:

$$\sum_{i_1=1}^{3} \sum_{i_2=1}^{3} \cdots \sum_{i_n=1}^{3} q_{1,i_1} q_{2,i_2} \cdots q_{n,i_n} T_{i_1,i_2,\cdots,i_n} = 0$$

- where: matching tensors $\mathbf{T}$ depend exactly on the projection matrices P_i

  n = 2: fundamental (essential) matrix

  n = 3: trifocal tensors

  n = 4: quadrifocal tensors

- Uses of matching tensors:
  - Matching constraints
  - Useful for motion estimation from image correspondences

Multi-view geometry for generic imaging model:

- Projection rays are represented by Plücker coordinates:
    - let $\mathbf{A}$ and $\mathbf{B}$ be any 2 points on a 3D line
    - Plücker coordinates can be defined as:

$$\mathbf{L} = \begin{pmatrix} A_4 B_1 - A_1 B_4 \\ A_4 B_2 - A_2 B_4 \\ A_4 B_3 - A_3 B_4 \\ A_3 B_2 - A_2 B_3 \\ A_1 B_3 - A_3 B_1 \\ A_2 B_1 - A_1 B_2 \end{pmatrix}$$

    - they are independent of the choice of $\mathbf{A}$ and $\mathbf{B}$

[Sturm-CVPR'05]

- Consider projection rays $\mathbf{L}_i$ for n calibrated cameras
- For the moment, parameterize rays by two points $\mathbf{A}_i$ and $\mathbf{B}_i$ each.
- Pose of cameras is parameterized as

$$\mathsf{P}_i = \begin{pmatrix} \mathsf{R}_i & \mathbf{t}_i \\ \mathbf{0}^\mathsf{T} & 1 \end{pmatrix}$$

- Rays are potential matches if scalars λ_i and μ_i and a 3D point $\mathbf{Q}$ exist with:

$$\lambda_i \mathbf{A}_i + \mu_i \mathbf{B}_i = \mathsf{P}_i \mathbf{Q}, \ \forall i = 1 \cdots n$$

- Rays are potential matches if scalars λ_i and μ_i and a 3D point $\mathbf{Q}$ exist with:

$$\lambda_i \mathbf{A}_i + \mu_i \mathbf{B}_i = \mathsf{P}_i \mathbf{Q}, \quad \forall i = 1 \cdots n$$

- This can be written as:

$$\underbrace{\begin{pmatrix} \mathsf{P}_1 & \mathbf{A}_1 & \mathbf{B}_1 & \cdots & \mathbf{0} & \mathbf{0} \\ \mathsf{P}_2 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathsf{P}_n & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_n & \mathbf{B}_n \end{pmatrix}}_{\mathsf{M}} \begin{pmatrix} \mathbf{Q} \\ -\lambda_1 \\ -\mu_1 \\ \vdots \\ -\lambda_n \\ -\mu_n \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix}$$

- Existence of null-vector implies rank-deficiency of M
- M is of size 4n × 4+2n
  → all submatrices (4+2n) × (4+2n) have zero determinant

$$\underbrace{\begin{pmatrix} \mathsf{P}_1 & \mathbf{A}_1 & \mathbf{B}_1 & \cdots & \mathbf{0} & \mathbf{0} \\ \mathsf{P}_2 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathsf{P}_n & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_n & \mathbf{B}_n \end{pmatrix}}_{\mathsf{M}}$$

- When developping determinants of submatrices, coordinates of points $\mathbf{A}_i$ and $\mathbf{B}_i$ appear in terms of this form:

$$A_{i,j}B_{i,k} - A_{i,k}B_{i,j}$$

$\rightarrow$ Plücker coordinates of $\mathbf{L}_i$

- We obtain matching constraints of the form:

$$\sum_{i_1=1}^{6}\sum_{i_2=1}^{6}\cdots\sum_{i_n=1}^{6} L_{1,i_1}L_{2,i_2}\cdots L_{n,i_n}T_{i_1,i_2,\cdots,i_n} = 0$$

- Matching tensors $\mathbf{T}$ depend on pose matrices P_i

- Like for perspective images, matching tensors exist for 2, 3, and 4 cameras
- Example: two views

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 & A_{1,1} & B_{1,1} & 0 & 0 \\ 0 & 1 & 0 & 0 & A_{1,2} & B_{1,2} & 0 & 0 \\ 0 & 0 & 1 & 0 & A_{1,3} & B_{1,3} & 0 & 0 \\ 0 & 0 & 0 & 1 & A_{1,4} & B_{1,4} & 0 & 0 \\ R_{11} & R_{12} & R_{13} & t_1 & 0 & 0 & A_{2,1} & B_{2,1} \\ R_{21} & R_{22} & R_{23} & t_2 & 0 & 0 & A_{2,2} & B_{2,2} \\ R_{31} & R_{32} & R_{33} & t_3 & 0 & 0 & A_{2,3} & B_{2,3} \\ 0 & 0 & 0 & 1 & 0 & 0 & A_{2,4} & B_{2,4} \end{pmatrix}$$

of size 8x8

M is rank-deficient, thus singular

$\rightarrow$ matching constraint is: $\det M = L_2^T \underbrace{\begin{pmatrix} -[t]_x R & R \\ R & 0 \end{pmatrix}}_{\text{essential matrix}} L_1 = 0$

- Matching tensors for non-central cameras are of size $6 \times 6 \times \ldots$
- Reduced parameterizations exist:
  - Axial cameras: $5 \times 5 \times \ldots$
  - X-slit cameras: $4 \times 4 \times \ldots$
  - Central cameras: $3 \times 3 \times \ldots$
- Matching tensors between cameras of different types are straightforward, e.g.:
  - Essential matrix of a non-central and a central camera: 6×3

Summary for structure-from-motion:

- When calibrated cameras are considered, an SfM problem (pose, motion, …) can be solved with one and the same algorithm, whatever the type of camera
- But: results are not optimal (e.g. in the sense of reprojection errors)
  → methods are useful for embedding in RANSAC, but should be followed by bundle adjustment if good accuracy required
- Extension of structure-from-motion theory from perspective to general camera model
- Some missing pieces, e.g. matching tensors for line images

Tutorial on

Modeling and Analysing Images of Generic Cameras

Peter Sturm, INRIA Rhône-Alpes, Montbonnot/Grenoble, France

http://perception.inrialpes.fr/people/Sturm

Bonn, September 19, 2006

With contributions from:

Rahul Swaminathan	Deutsche Telekom Laboratories TU Berlin
Srikumar Ramalingam	INRIA Rhône-Alpes and UC Santa Cruz
Jean-Philippe Tardif	Université de Montréal

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